Solutions to Review Problems for Differential Equations: Take Two

1. Graph the differential equation with $\frac{dw}{dt}$ on the vertical axis and w on the horizontal axis. Get a parabola that opens upward with zeroes at $w = 4$ and $w = 6$. For $f < w < 6$, $\frac{dw}{dt} < 0$; thus the solutions $w(t)$, must be decreasing for $4 < w < 6$. Similarly, for $w < 4$ or $w > 6$, the solutions, $w(t)$ must be increasing. Answer: (d).

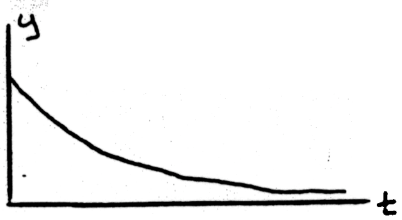
2. The solutions are always increasing. Thus, we must have $\frac{dP}{dx} \geq 0$. Answer: (e). Note: (d) and (f) have $\frac{dP}{dx} < 0$ for negative x. (b) and (c) have $\frac{dP}{dx} < 0$ for $P < 0$. (a) has $\frac{dP}{dx} < 0$ for $P > 0$.

3. Do a rough phase plane analysis: $\frac{dx}{dt} = -2$, $\frac{dy}{dt} = 4x$ (Answer: 5)

$\frac{dx}{dt} = 3y$, $\frac{dy}{dt} = -3x$ (Answer: 9)

$\frac{dx}{dt} = 10x$, $\frac{dy}{dt} = 10y$ (Answer: 7)

4. Differential equation: $\frac{dy}{dt} = -ky$. Solution: $y = Ce^{-kt}$.



5. (a) Let $y(t)$ = bank balance at time t. Then $\frac{dy}{dt} = -12,000 + .06y$. If the initial amount is 200,000, then $\frac{dy}{dt} = 0$ and y will remain constant.

(b) Solve by separating variables:

$$\int \frac{dy}{-12,000 + .06y} = \int dt.$$

$$\frac{1}{.06} \ln |.06y - 12,000| = t + c.$$

After a few more steps, you get $y(t) = 200,000 + Ae^{.06t}$. If $y(0) = y_0$, then $y = 200,000 + (y_0 - 200,000)e^{.06t}$. To have $y(20) = 0$, we must have $y(0) \approx 140,000$.

6. At $(1,1)$, $\frac{dx}{dt}$ is positive. This is only the case in (c).

7. (a) $\frac{dT}{dt} = \frac{1}{29}(10 - T)$

(b) $\frac{dT}{dt} = \frac{10 - T}{29} \Rightarrow t = -29\ln(10 - T) + C \Rightarrow t - C = -29\ln(10 - T) \Rightarrow \ln(10 - T) = -\frac{t}{29} + k \Rightarrow 10 - T = Ke^{-\frac{t}{29}} \Rightarrow T = 10 + Ke^{-\frac{t}{29}}$. From the initial value $T(0) = 65$, we get $K = 55$, so $T = 10 + 55e^{-\frac{t}{29}}$. Therefore, $T(5) \approx 56.3$.

(c) $\frac{dT}{dt} = 2 + \frac{1}{29}(10 - T)$

$(2 + \frac{1}{29}(10 - T) = 0$ for $T = 68)$

(d) Independent of the initial temperature, the temperature will approach 68°.

8. Look at the characteristic eq'n : $r^2+r+4r=0$ $\Rightarrow r = -\frac{1}{2} \pm \frac{\sqrt{15}}{2} i$

So $x(t) = e^{-\frac{1}{2}t}\left[C_1 \cos \frac{\sqrt{15}}{2} t + C_2 \sin \frac{\sqrt{15}}{2} t\right]$ $\Rightarrow$ only (a) or (c) is possible.

Only (a) has $x(0)<0)$ & $x'(0)<0$.

9. Characteristic eq'n: $r^2-r+1=0$ so $r=\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$; $y(t) = e^{t/2}\left[C_1 \cos \frac{\sqrt{3}}{2} t + C_2 \sin \frac{\sqrt{3}}{2} t\right]$

$e^{t/2}$ increases with t so the answer must be (a).

10. h
11. c
12. b
13. a) 5 b) 2
14. a. C Only C is always concave up

b. A $y=-e^{t}+2$, $y'<0$ if $y<2$

c. B Zero sol'n: $y=0$

d. D $y=e^{-t^2/2}$