Solutions to Integration Problems

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1. Perform integration by parts with $u = x, dv = \sec^2 x dx$. The answer is $x \tan x + \ln |\cos x| + C$

2. Perform the substitution $u = 2x + 1$. The answer is $2x + \ln |2x + 1| + C$

3. Perform the substitution $u = x^2$. The answer is $\frac{7}{2} e^{x^2} + C$

4. Use the fact that $x^2 + 6x + 9 = (x+3)^2$. The answer is $-\frac{1}{x+3} + C$

5. Since $\sin(x^3)$ is an odd function and the interval is symmetric around 0, the integral is zero. (Note that we would not want to find an antiderivative for this function.) 0

6. Perform the substitution $u = 3x + 4$. The answer is $\frac{2}{9}(31\sqrt{31} - 7\sqrt{7})$

7. Integrate by parts with $u = \tan^{-1} x$ and $dv = x$. Complete the integration by using long division. The answer is $-\frac{x}{2} + \frac{\tan^{-1} x}{2} + \frac{1}{2} x^2 \tan^{-1} x + C$.

8. Integrate by parts twice and note that we obtain the original integrand. The answer is $\frac{1}{2} e^x (\sin x - \cos x) + C$

9. Integrate by parts with $u = \ln x, dv = x dx$. The answer is $\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$

10. Use the trigonometric identity $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$. The answer is $\frac{1}{2} x - \frac{1}{28} \sin(14x) + C$

11. Perform the substitution $u = x^4 + x^3 + x^2 + x + 1$. The answer is $\ln |x^4 + x^3 + x^2 + x + 1| + C$

12. Use partial fractions. The answer is $\frac{9}{2} \ln |x - 1| - 15 \ln |x - 2| + \frac{21}{2} \ln |x - 3| + C$

13. $\frac{1}{2} \sin(x^2) + C$.   Substitute for x^2.

14. $x^2 \sin x + 2x \cos x - 2 \sin x + C$.   Integrate by parts twice.

15. $\frac{1}{2} \ln |x - 1| + \frac{3}{2} ln|5 + x| + C$. Integrate by parts after factorizing the denominator.

16. $\frac{1}{2} x + \frac{1}{4} \sin(2x) + C$.   Use trig identity $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$

17. $\frac{1}{2} x^2 \sin(x^2) + \frac{1}{2} \cos(x^2) + C$.   Substitute for x^2, and then use parts - or just use parts with $u = x^2$ and $dv = x \cos(x^2)$.

18. $\sin(x) - \frac{1}{3} \sin^3(x) + C$.   Use trig identity $\cos^2 x = 1 - \sin^2 x$ by rewriting integrand as $\cos^2(x) \cos(x)$. Substitute for $\sin^2 x$.

19. 1.   Integrate by parts.

20. $x(\ln x)^2 - 2x \ln x + 2x + C$.   Integrate by parts twice.

21. $\frac{1}{3}(\ln x)^3 + C$.   Substitute for $\ln x$

22. $\ln 3$.   Substitute $u = \ln x$.

23. $\frac{1}{2}(x^2 + 1) \ln(x^2 + 1) - \frac{1}{2}(x^2 + 1) + C$.   Substitute for x^2 and then do by parts. Your answer may differ by a constant, of course.

24. $x - 2 \arctan x + C$.   Realize that the fraction equals $1 - \frac{2}{x^2+1}$. You can also use long division to do this.

25. $x - \ln |x + 1| + \ln |x - 1| + C$.   Note $1 + \frac{2}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}$. Note that long division could be used here as well.

26. $\frac{1}{2} \tan^{-1}(x^2) + C$.

27. $\sqrt{x^2 + 1} + C$.   Substitute $u = x^2 + 1$.

28. $\frac{1}{4} \ln |e^x - 2| - \frac{1}{4} \ln |e^x + 2| + C$.   Substitute for e^x followed by partial fractions

29. $\frac{1}{2} \ln |e^{2x} - 4| + C$.   Substitute for e^{2x}.

30. $2\sqrt{7} - 2\sqrt{5}$.   Substitute for $e^x + 4$.

31. $2 \ln |x - 3| - \ln |x + 1| + C$.   Partial fractions.

32. $x + 2 \tan^{-1} x + C$.   Realize that the fraction equals $1 + \frac{2}{x^2+1}$.

33. $\frac{1}{3} e^{x^3} + C$.   Substitute for x^3.

34. $-\frac{3}{2} \ln |\cos 2x| + C$.   Write as $\frac{\sin x}{\cos x}$ and then substitute for $\cos x$.

35. $e^{\tan x} + C$.   Substitute for $\tan x$.

36. $-\frac{\cos(2 \sin x)}{2} + C$.   Substitute for $\sin x$.

37. $\frac{\ln(1+x)}{(1+x)} + \frac{1}{(1+x)} + C.$ Let $u = x + 1$ and then integrate by parts.

38. $-\frac{2}{1+x} + C.$ Factor the denominator and look at the problem.

39. $\frac{1}{2} \ln \frac{x}{x+2} + C.$ Partial Fractions.

40. $\frac{3}{4} \ln |x - 1| - \frac{1}{2(x-1)} + \frac{1}{4} \ln |x + 1| + C.$ Partial fractions.

41. $3x \sin x + 3 \cos x + C.$ Integrate by parts.

42. $\frac{1}{3} (x^2 + 2x)^{\frac{3}{2}} + C.$ Write as $(x+1)\sqrt{x^2 + 2x}$. Substitute for $x^2 + 2x$.

43. $1 - \cos(1).$ Substitute for $\ln x$.

44. $\frac{x}{2} \left(\sin(\ln(x)) - \cos(\ln(x)) \right) + C.$ Integrate by parts twice, look to switch terms over.

45. $\frac{-1}{3x} - \frac{\arctan(x/3)}{9}.$ Partial Fractions.

46. $\frac{x^2}{2} - \frac{1}{2} \ln(x^2 + 1) + C.$ Write fraction as $\frac{x(x^2+1)-x}{x^2+1}$. Split the fraction and split the integral across the minus sign. Substitute for x^2 on the right side. Another way would to be approach via long division.

47. $\ln |x| - \frac{1}{2} \ln |x^2 + 1| + C.$ Partial Fractions.

48. $\frac{1}{3} \ln \left| \frac{1-\cos(x)}{4-\cos(x)} \right| + C.$ Substitute for $\cos x$. Partial fractions.

49. $2e^{\sqrt{x}}.$ Substitute for $\sqrt{x}$.

50. $2e^{-x}(-x - 1).$ Integrate by parts with $u = 2x$ and $dv = e^{-x}$.

51. $\frac{1}{2} \ln(x^2 + 1) + \tan^{-1}(x).$ Write the fraction as $\frac{x}{x^2+1} + \frac{1}{x^2+1}$.

52. $e^{(x+1)^2}.$ Write as $(2x + 2)e^{(x+1)^2}$. Substitute for $(x + 1)^2$.

53. $\frac{2}{x} - \ln |x-1| + \ln |x|.$ Integrate by partial fractions. Remember that the x^2 in the denominator adds both an x^2 and an x factor to the partial fraction method.

54. $-\sqrt{1 - x^2}.$ Substitute $u = 1 - x^2$.

55. $-\frac{1}{2} \cos(x^2 - 1).$ Substitute for $x^2 - 1$.

56. $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}.$ Factor $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$. Obtain this through long division.

57. $\frac{(x^2+1)^5}{5} + \frac{2(x^2+1)^3}{3} + x^2 + 1.$ Substitute $u = x^2 + 1$.

58. $0.$ Realize $\frac{x^3}{\sqrt{1-x^2}}$ is an odd function.

59. $2e^3/9 + 1/9.$ Integrate by parts

60. $-\frac{1}{4} \cos(2x) + C.$ Substitute $u = \cos(x).$

61. $\ln \ln \ln x + C.$ Substitute $u = \ln(x).$

62. $-\frac{1}{2} \cos 2x + \frac{1}{6} \cos^3(2x) + C$ Substitute $u = 2x$. Remember $\sin^2(u) = 1 - \cos^2(u).$

63. $x \ln \sqrt{x} - x/2 + C$ Remember $\ln(x^a) = a \ln(x)$. Integrate by parts.

64. $\frac{3}{4} \ln |x - 2| + \frac{1}{4} \ln |x + 2| + C.$ Partial Fractions.

65. $e^{e^x} + C.$ Note that $e^{e^x+x} = e^{e^x}e^x$. Substitute $u = e^x.$

66. $\ln(1 + \sin(x)).$ Substitute $u = 1 + \sin(x).$

67. $\frac{x^2}{2} + \ln |x - 1|.$ You can simplify the integrand to $x + \frac{1}{x-1}$ using long division followed by factoring.

68. $\frac{\ln |x|}{a^2} - \frac{\ln |x^2+a^2|}{2a^2}.$ Long division followed by Partial Fractions.

69. $-(x^2 - a^2)^{-0.5} + C.$ Substitute $u = x^2 - a^2$

70. $\frac{x^2(\ln x)^2}{2} - \frac{x^2}{2} \ln x + \frac{x^2}{4} + C.$ Integrate by parts twice.

71. $3 \left(\ln 2 - \frac{1}{2} \right).$ Substitute $u = x^{\frac{1}{3}}$. Then $u^3 = x$ so $3u^2 du = dx.$

72. $\tan x - x + C.$ Use Trig. Identity $\tan^2(u) = \sec^2(u) - 1.$

73. $\frac{\pi}{12}.$ Note that the integrand can be written as $\frac{1}{e^x + \frac{1}{e^x}}$ which in turn equals $\frac{e^x}{e^{2x}+1}$. Now Substitute $u = e^x$. Look for the integral of ArcTan.

74. $-\frac{1}{36}.$ Integrate by parts.

75. $\frac{1}{\sqrt{e}} \left(1 - \frac{1}{\sqrt{e}} \right).$ Substitute $u = -\frac{1}{x}.$

76. $\sqrt{x^2 + 3} + C.$ Substitute $u = x^2 + 3.$

77. $x^2/2 + C.$ Realize $e^{\ln x} = x.$

78. $1/2 \ln(x^2 + 1) - \ln |x + 1| + C.$ Partial Fractions.

79. $t \ln t - t + C$, where $t = \sin x.$ Substitute $u = \sin x.$ Then integrate by parts.

80. $2(t - \arctan(t)) + C$, where $t = \sqrt{x}.$ Substitute $u = \sqrt{x}$ so $u^2 = x$ and $2u du = dx.$

81. $(2/3)t^{3/2} + C$, where $t = \ln x.$ Substitute $u = \ln x.$

82. $(x^2 - 2x + 2)e^x + C.$ Integrate by parts twice.

83. $x \ln(x^2 + 1) - 2x + 2 \arctan(x) + C.$ Integrate by parts.

84. $t \ln t - t + C$, where $t = \ln x.$ Substitute $u = \ln x.$ Then by parts.

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