First Exam for Mathematics 1b

March 4, 2003

You have two hours for this exam. Work carefully and efficiently. Do not spend an inordinate amount of time on any one problem. Think clearly and do well!

Problem	Points	Score
1	20	
2	14	
3	15	
4	18	
5	18	
6	15	
Total	100	

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get full credit. If you have work on the back of a page, indicate that on the exam cover.

Please circle your section.

MWF 10	Amanda Alvine	MWF 11	Dylan Thurston	MWF 12	Peter Kronheimer
TTH 10	Robin Gottlieb	TTH 11:30	Alex Popa	TTH 11:30	Robin Gottlieb

1. (20 points) For each of the following improper integrals, either evaluate the integral or determine that it diverges.

   (a) $\displaystyle\int_0^\infty \frac{x}{1+x^2}\,dx$

   (b) $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos x}{\sin x}\,dx\ 1\frac{}{\sqrt{1-x}\,dx}$

   (c) The improper integral that gives the volume obtained by revolving around the x-axis the region bounded below by the $x-$axis, on the left by the $x = e$ and above by $y = \dfrac{1}{\sqrt{x}\,(\ln x)}$.

2. (14 points) The compound capsaicin is responsible for the heat in chilli peppers. Joe has run out of Habañero peppers at his popular pizza restaurant, and is therefore spraying pure capsaicin, dissolved in olive oil, on his Greenhouse specials.

   His pizzas have a 15 cm radius, and his spray delivers a density of capsaicin that varies with the distance from the center of the pizza: the density of capsaicin at distance x cm from the center is given by the formula:

   $$\text{density} = 3\,e^{-x^2/100}\ \text{mg}/\text{cm}^2.$$

   How much capsaicin is on each pizza?

3. (15 points) An ant-hill in the Amazon rainforest has the shape of a perfect hemisphere, with its base on the forest floor. It has a $1\,\mathrm{m}$ radius, and a uniform density of $50\,\mathrm{kg}/\mathrm{m}^3$. How much work have the ants done to assemble this ant-hill, lifting materials from the level of the forest floor?

   (Take the value of g to be $10\,\mathrm{m}/\mathrm{s}^2$.)

4. (18 points)

   (a) An elegant solid bronze hard boiled egg holder is constructed by rotating the region bounded by the $x-$axis, the $y-$axis, the line $x = 2$, and $y = x^2 + 1$ about the $y-$axis. Here all measurements are given in inches. Write an integral that gives the number of cubic inches of molten bronze required for the construction of the egg holder. You need not compute the integral.

   (b) A jello mold sitting in a refrigerator has the following form. The area above the graph of $y = \frac{3}{4}x^2$ and below the line $y = 3$ is rotated around $x = 5$. Here all measurements are given in inches. The jello mold is filled to the brim with jello into with we've mixed pulverized pears. The pulverized pears tend to sink to the bottom of the mold. The density of pears is given by $g(y)$ ounces per cubic inch where y is the number of inches from the refrigerator shelf. (We have *not* turned the mold over. This y agrees with the y given to describe the mold.) Write an integral that gives the number of ounces of pulverized pears that were used in the jello. You need not compute the integral.

5. (18 points) A bucket that can hold 10 lb of water is raised from the ground to the top of a building 200 ft high, using a rope having a linear density of 0.1 lb/ft. Initially the bucket is half full (contains 5 pounds of water), but heavy rain fills it up at a constant rate so that it is full exactly when it reaches the top of the building. Assuming the bucket is raised at a constant velocity, how much work is required to bring it to the top?

6. (15 points)

   (a) (6 points) Suppose $f(x) \geq 0$ for $x \geq 1$, where $f(x)$ is continuous and defined for all $x \geq 1$ and $\lim_{x\to\infty} f(x) = 0$. Which of the following is true?

      i. $\int_1^\infty f(x)\,dx$ must converge.
      ii. $\int_1^\infty f(x)\,dx$ must diverge.
      iii. $\int_1^\infty f(x)\,dx$ could converge or could diverge.

      If you choose one of the first two choices, explain your reasoning clearly. If you choose the last, give examples of each scenario.

   (b) (6 points) Suppose $0 < f(x) < 3$ for $x \geq 1$, where $f(x)$ is continuous and defined for all $x \geq 1$ and $\lim_{x\to\infty} f(x) = .000001$. Which of the following is true?

      i. $\int_1^\infty f(x)\,dx$ must converge.
      ii. $\int_1^\infty f(x)\,dx$ must diverge.
      iii. $\int_1^\infty f(x)\,dx$ could converge or could diverge.

      If you choose one of the first two choices, explain your reasoning clearly. If you choose the last, give examples of each scenario.

   continued

   (c) (3 points) Suppose you know that $0 < f(x) < 1$ and f is continuous and defined for all x and that $\int_1^\infty f(x)\,dx$ converges. Which of the following is true?

      i. $\int_1^\infty [f(x)]^3\,dx$ must converge.
      ii. $\int_1^\infty [f(x)]^3\,dx$ must diverge.
      iii. $\int_1^\infty [f(x)]^3\,dx$ could converge or could diverge.

   Explain your reasoning.