

## Integration Techniques

In each problem, decide which method of integration you would use. If you would use substitution, what would  $u$  be? If you would use integration by parts, what would  $u$  and  $dv$  be? If you would use partial fractions, what would the partial fraction expansion look like? (Don't solve for the coefficients.)

1.  $\int \frac{\cos x \, dx}{\sqrt{1 + \sin x}}.$

**Solution.** Use the substitution  $u = 1 + \sin x$  since  $du = \cos x \, dx$  also appears in the integrand.

2.  $\int (\ln x)^2 \, dx.$

**Solution.** Partial fractions is definitely not right, since this is not a rational function. Substitution doesn't look so promising, so we're left with integration by parts. Since we don't know how to integrate anything involving  $\ln$ , use  $u = (\ln x)^2$  and  $dv = dx$ .

3.  $\int e^x \sin x \, dx.$

**Solution.** This is a classic integration by parts integral, where you do integration by parts twice to get back the original integral and then solve for it. You can use  $u = e^x$  and  $dv = \sin x \, dx$  or  $u = \sin x$  and  $dv = e^x \, dx$ ; they work equally well.

4.  $\int \frac{x}{x^2 - 1} \, dx.$

**Solution.** Since the integrand is a rational function, you could use partial fractions. But it's easier to just use substitution with  $u = x^2 - 1$ .

5.  $\int x e^{x^2} \, dx.$

**Solution.** Substitution with  $u = x^2$  since  $du = 2x \, dx$  also appears.

6.  $\int \frac{x^2}{x^2 + 4x + 3} \, dx.$

**Solution.** The integrand is a rational function, and we can factor the denominator pretty easily, so partial fractions is a good choice. Since the integrand is an improper fraction, we start by rewriting it:  $\frac{x^2}{x^2+4x+3} = \frac{x^2+4x+3-4x-3}{x^2+4x+3} = 1 - \frac{4x+3}{x^2+4x+3}$ . The denominator factors as  $(x+1)(x+3)$ , so the partial fraction expansion has the form  $1 + \frac{A}{x+1} + \frac{B}{x+3}$ .

7.  $\int \frac{e^t}{1 + e^t} \, dt.$

**Solution.** Use substitution with  $u = 1 + e^t$  since  $du = e^t \, dt$  also appears.

8.  $\int \arcsin x \, dx.$

**Solution.** Partial fractions is definitely out, and there's not much to substitute, so use integration by parts with  $u = \arcsin x$  and  $dv = dx$ .