

Density and Slicing

1. A seaside village, Playa del Carmen, is in the shape of a rectangle 4 miles wide and 6 miles long. The sea lies along a 6-mile long side. People prefer to live near the water, so the density of people is given by $\rho(x) = 10000 - 800x$ people per square mile, where x is the distance from the seaside. We would like to find the population of the village.

(a) Show in a sketch how to slice up the region.

(b) What is the area of the k -th slice?

(c) What is the approximate population in the k -th slice?

(d) Write a general Riemann sum to estimate the total population of the city.

(e) Find a definite integral expressing the population of the village.

2. A rectangular plot of farm land is 300 meters by 200 meters. A straight irrigation pipe 300 meters long runs down the center of the plot, dividing it in half lengthwise. The farmer's yield decreases as the distance from the irrigation pipe increases. Suppose that the yield is given by $\rho(x)$ grams per square meter, where x is the distance in meters from the irrigation pipe. Write an integral giving the total yield from the plot.

3. People in the Boston area like to live near the city center, so the population density around Boston is $\rho(r) = \frac{36,000}{r^2+2r+1}$ people per square mile, where r is the distance in miles to the center of Boston. Find the number of people who live within 5 miles of the center of Boston.