

Improper Integrals

An integral is improper if

1. the interval of integration is unbounded and/or
2. the integrand is unbounded somewhere on the interval of integration

The basic strategy for dealing with an improper integral is:

1. Identify all improprieties.
2. If necessary, split up the integral into a sum so that every impropriety is an endpoint of integration and each piece of the sum has at most one impropriety.
3. Compute each of these improper integrals as a limit of definite integrals.
4. If any one summand (piece) diverges, the whole thing diverges. If every piece converges, sum to find out what the original converges to.

⚡ **Caution:** This means that divergent summands can never “cancel” one another!

Determine whether the following integrals converge or diverge. Explain your reasoning.

1. $\int_{-2}^2 \frac{x}{x^2 - 1} dx.$

2. $\int_1^{\infty} \frac{1}{x^4 + 2} dx.$

3. $\int_0^{\infty} \frac{1}{e^x + x} dx.$

4. $\int_{-\infty}^{\infty} \sin x dx.$

5. $\int_1^{\infty} \frac{1 + e^{-x}}{x} dx.$

6. $\int_1^{\infty} \frac{\cos^2 x}{x^2} dx.$