

Probability

Waiting times, shelf-lives, and equipment failure times are often modeled by exponentially decreasing probability density functions.

1. Suppose $f(t) = 0$ for $t < 0$ and $f(t) = 0.5e^{-ct}$ for $t \geq 0$ is the probability density function for the lifetime of a particular toy (t in years).

(a) For what value of c is this a probability density function?

(b) What is the probability that the toy lasts over one year? (Is there any way to compute this without computing an improper integral?)

(c) What is the median life of this type of toy?

2. A large number of students take an exam. 30% of the students receive a score of 70, 50% receive a score of 80, and 20% receive a score of 90. What is the average score on the exam?

3. The density function for the duration of telephone calls within a certain city is $p(x) = 0.4e^{-0.4x}$ where x denotes the duration in minutes of a randomly selected call.
- (a) What percentage of calls last one minute or less?
- (b) What percentage of calls last between one and two minutes?
- (c) What percentage of calls last 3 minutes or more?
- (d) What is the average length of a call?
4. The lifetime, in hundreds of hours, of a certain type of light bulb has been found empirically to have a probability density function approximated by $f(x) = \frac{\sqrt{65}}{8(1+x^2)^{3/2}}$ for $0 < x < 8$. Find the mean lifetime of such a bulb.