Geometric Sums and Geometric Series

1. In your quest to become a millionaire by age 50, you start an aggressive savings plan. You open a new investment account on January 1, 2008 and deposit \$9000 into it every year on January 1. Each year, you earn 7% interest on December 31.

   (a) How much money will you have in your account on January 2, 2009? 2010? 2014? (Don't try to add or multiply things out; just write an arithmetic expression.)

   (b) Will you be a millionaire by age 50?

2. If you suffer from allergies, your doctor may suggest that you take Claritin once a day. Each Claritin tablet contains 10 mg of loratadine (the active ingredient). Every 24 hours, about 7/8 of the loratadine in the body is eliminated (so 1/8 remains). [1]

   (a) If you take one Claritin tablet every morning for a week, how much loratadine is in your body right after you take the 3rd tablet? 7th tablet? (Don't try to simplify your computations; just write out an arithmetic expression.)

[1] This estimate comes from the fact that the average half-life of loratadine is known to be 8 hours.

(b) If you take Claritin for years and years, will the amount of loratadine in your body level off? Or will your bloodstream be pure loratadine?

3. For what values of x does the geometric series $1 + x + x^2 + \cdots$ converge? [2] If it converges, what does it converge to?

4. Which of the following series are geometric?

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 2^{k+1}}{3^k}$.

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3^{n/2}}$.

[2] We could also write this series in summation notation as $\displaystyle\sum_{k=0}^{\infty} x^k$.