Let $a_1 + a_2 + a_3 + \cdots = \sum_{k=1}^{\infty} a_k$ be an infinite series, and let s_n denote its n-th partial sum: $s_n = a_1 + a_2 + a_3 + \cdots + a_n$.

- If $\lim_{n\to\infty} s_n = s$ for a finite number s, we say the series $a_1 + a_2 + a_3 + \cdots$ converges (to s) and write $a_1 + a_2 + a_3 + \cdots = s$.
- If $\lim_{n\to\infty} s_n$ does not exist (or is not finite), we say the series $a_1 + a_2 + a_3 + \cdots$ diverges.

1. Suppose you know that the infinite series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ converges to s and that $a_k > 0$ for k any positive integer. Let $s_n = a_1 + a_2 + a_3 + \cdots + a_n$. For each of the following statements, determine whether the statement must be true, could possibly be true, or must be false.

   (a) $\lim_{n\to\infty} a_n = 0$.

   (b) $\lim_{n\to\infty} s_n = 0$.

   (c) There exists a number M such that $s_n < M$ for all n. (This is equivalent to saying that the partial sums are bounded. Why?)

   (d) $\sum_{k=5}^{\infty} a_k$ converges.

2. Suppose you know that $\lim_{n\to\infty} b_n = 0$. Can you be sure that the infinite series $b_1+b_2+b_3+\cdots$ converges?

A sequence is an ordered list of numbers. Some examples are:

- $-2, -4, -6, -8, \ldots$
- $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots$

Sometimes sequences are also written in the form $\{s_n\}$ where s_n is the n-th term of the sequence. For instance, the two examples above are $\{-2n\}$ and $\left\{\left(-\frac{1}{2}\right)^n\right\}$.

3. (a) Give an example of a sequence (ordered list) of numbers such that the numbers are increasing but are bounded.

   (b) Give an example of a sequence (ordered list) of numbers such that the numbers are increasing and are not bounded.

   (c) Give an example of a sequence (ordered list) of numbers such that the numbers are bounded but have no limit as $n \to \infty$.

A sequence is monotonic if it is either increasing (each term is greater than or equal to the preceding term) or decreasing (each term is less than or equal to the preceding term).

4. (a) A sequence which is both monotonic and bounded

   must converge   could either converge or diverge   must diverge

   (b) A sequence which is monotonic but not bounded

   must converge   could either converge or diverge   must diverge

5. Consider the series $\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ (called the harmonic series).

   (a) Does the sequence of terms converge? If so, to what does it converge?

   (b) Does the sequence of partial sums converge? If so, to what does it converge?

   (c) Does the series $\sum_{k=1}^{\infty} \frac{1}{k}$ converge?

   (d) Does the series $\sum_{k=10^{10}}^{\infty} \frac{1}{100000000k}$ converge?

6. See if you can determine whether each of the following series converges or diverges by using the Nth Term Test for Divergence, results about geometric series, or some sort of comparison to series you know about.

   (a) $\sum_{k=100}^{\infty} \frac{1}{3k}$.

   (b) $\sum_{k=3}^{\infty} \frac{(-1)^k 2^k}{3^k}$.

   (c) $\sum_{k=4}^{\infty} \frac{(-1)^k 3^k}{2^k}$.

   (d) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

   (e) $\sum_{n=2}^{\infty} \frac{n}{\ln n}$.

   (f) $\sum_{n=0}^{\infty} \sin n$.