Comparison

1. Use the Comparison Test (also known as "direct comparison") to decide whether the following series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}3^n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3 - n + 1000}$.

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln(1+n)}$.

2. True or false: If $\{a_n\}$ is a sequence with positive terms and $\lim_{n\to\infty} a_n = 0$, then there is a number k such that $a_n < 1$ whenever $n \geq k$.

3. Decide whether the following series converge or diverge using any method you like.

(a) $\displaystyle\sum_{n=100}^{\infty} \cos n$.

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 2^{k+1}}{3^k}$.

(c) $1 + 0 + (-1) + 1 + 0 + (-1) + 1 + 0 + (-1) + \cdots$.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$.