More Comparison

1. For what values of p is the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent?

2. Does the series $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$ converge or diverge?

3. Does the series $\sum_{k=1}^{\infty} \frac{3}{8^k - 2}$ converge or diverge?

4. Otto is given the following problem for homework.

   Decide whether the series $\sum_{n=1}^{\infty} \sin^2(\pi n)$ converges or diverges. Explain your reasoning.

   Otto writes

   The improper integral $\int_1^{\infty} \sin^2(\pi x)\, dx$ diverges, so $\sum_{n=1}^{\infty} \sin^2(\pi n)$ also diverges by the Integral Test.

   Otto is correct that the improper integral diverges (although he should have shown more work!). But the rest of his reasoning is incorrect — why? And what is the correct answer to the problem?

5. Let $\sum a_n$ and $\sum b_n$ be series with positive terms. The Limit Comparison Test only applies when $\lim_{n\to\infty} \frac{a_n}{b_n}$ is a positive real number.

   (a) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, can you draw any conclusions?

   (b) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, can you draw any conclusions?