Ratio Test

1. What does the Ratio Test tell you about the following series?

   (a) $\displaystyle\sum_{k=0}^{\infty}(-1)^{k+1}\frac{1000^k}{k!}$.

   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k}$.

   (c) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2}$.

2. When we studied Taylor series, we found that the Taylor series for $\sin x$ about 0 was $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$, which can be written in summation notation as $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!}$. For what values of x does this series converge?

3. When we studied Taylor series, we found that the Taylor series for $\ln(1+x)$ about 0 was $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$, which can be written in summation notation as $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}$. For what values of x does this series converge?

4. Decide whether the following series converge absolutely, converge conditionally, or diverge. You may use any method you like, but explain your reasoning. There is one that you will not be able to do (this is not due to a personal failing; it's just that all of the tests that we know are inconclusive).

(a) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$.

(b) $\sum_{n=100}^{\infty} \frac{n!n!}{(2n)!}$.

(c) $\sum_{n=0}^{\infty} \frac{\sin n}{n}$.

(d) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$.

(e) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$.

(f) $\sum_{n=5000}^{\infty} (-1)^n \frac{n}{n+1}$.