# Series Problems

1. Decide whether the following series converge or diverge. Explain your reasoning.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^5 + 500n^4 + n^3}{n^7 + 200n^6}$.

   (b) $\displaystyle\sum_{n=100}^{\infty} \frac{\sin n}{n^2}$.

2. Use a second degree Taylor polynomial to approximate $33^{1/5}$.

3. We define a function $f(x)$ by setting $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{\sqrt{n}2^n}$ for those $x$ for which the series converges.

(a) Find the radius of convergence.

(b) Write a power series representation of $f'(x)$, the first derivative of $f$. Use it to find a series for $f'(1)$.

(c) Write out the first three non-zero terms of your series for $f'(1)$. At $x = 1$, is the function $f$ increasing or decreasing? Explain.