

## Series Problems

1. Decide whether the following series converge or diverge. Explain your reasoning.

(a) 
$$\sum_{n=1}^{\infty} \frac{2n^5 + 500n^4 + n^3}{n^7 + 200n^6}.$$

(b) 
$$\sum_{n=100}^{\infty} \frac{\sin n}{n^2}.$$

2. Use a second degree Taylor polynomial to approximate  $33^{1/5}$ .

3. We define a function  $f(x)$  by setting  $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{\sqrt{n} 2^n}$  for those  $x$  for which the series converges.

(a) Find the radius of convergence.

(b) Write a power series representation of  $f'(x)$ , the first derivative of  $f$ . Use it to find a series for  $f'(1)$ .

(c) Write out the first three non-zero terms of your series for  $f'(1)$ . At  $x = 1$ , is the function  $f$  increasing or decreasing? Explain.