Differential Equations: An Introduction to Modeling

In #1 - #8, write a differential equation that reflects the situation. Include an initial condition if the information is given.

1. The population of a certain country increases at a rate proportional to the population size. Let $P = P(t)$ be the population at time t.

2. A snowball melts at a rate proportional to its surface area. At time 0, the snowball has a radius of 10 cm. Let $r = r(t)$ be the radius of the snowball at time t.

3. A yellow rubber duck is dropped out of the window of an apartment building at a height of 80 feet. Let $s = s(t)$ be the height of the duck above the ground at time t. (Gravity is the acceleration -32 ft/s^2.)

4. Ferdinand is trying to fill a bucket from a faucet. Unfortunately, he doesn't realize that there is a small hole in the bottom of the bucket. Water flows in to the bucket from the faucet at a constant rate of .75 quarts per minute, and it flows out of the hole at a rate proportional to the amount of water $W(t)$ already in the bucket (due to the increased water pressure).

5. A drug is being administered to a patient at a constant rate of c mg/hr. The patient metabolizes and eliminates the drug at a rate proportional to the amount in his body. Let $M = M(t)$ be the amount (in mg) of medicine in the patient's body at time t, where t is measured in hours.

6. $6000 is deposited in a bank account. The account has a nominal annual interest rate of 2%, compounded continuously. There are no deposits and no withdrawals. Let $M = M(t)$ be the amount of money in the account at time t, where t is measured in years.

7. $6000 is deposited in a bank account. The account has a nominal annual interest rate of 2%, compounded continuously. Money is being withdrawn at a rate of $500 per year.[1] Let $M = M(t)$ be the amount of money in the account at time t, where t is measured in years.

8. A rumor spreads at a rate proportional to product of the number of people who have heard it and the number who have not. In a town of N people, suppose 1 person originates the rumor at time $t = 0$. Let $y = y(t)$ be the number of people who have heard the rumor at time t.

   What does this model imply about the number of people who eventually have heard the rumor?

The following problems are about solutions to differential equations.

9. Which of the following is a solution to $\frac{dy}{dx} = y$?

   (a) $y = \frac{x^2}{2} + C$.

   (b) $y = e^x + C$.

   (c) $y = Ce^x$.

10. Give two solutions to $\frac{dy}{dx} = 5y$. What is the general solution?

11. Give two solutions to to $\frac{dy}{dx} = 5x$. What is the general solution?

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[1] In reality, you cannot withdraw money continuously from the bank, but it's convenient to use a continuous model.

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