Separation of Variables / Mixing Problems

1. Find the general solution of the differential equation $\frac{dM}{dt} = 2.4 - .2M$. (Such a differential equation came up, for instance, when we modeled the amount of medicine in a patient's body.)

2. Last time, we solved the differential equation $\frac{dy}{dt} = -\frac{t}{y}$ by drawing the slope field, guessing the solution, and checking it. Now, solve the differential equation using separation of variables.

3. Solve the differential equation $\frac{dy}{dt} = e^{-t-y}$, and find the particular solution satisfying the initial condition $y(0) = 1$.

4. Solve the differential equation $y' = 2y - 6$.

5. Which of the following differential equations are separable? (You need not solve.)

   (a) $\frac{dy}{dt} = t + y$.

   (b) $\frac{dy}{dt} = \frac{y}{\sin t}$.

   (c) $\frac{dy}{dt} = \frac{\sin t}{y} + t$.

6. A 20-quart juice dispenser in a cafeteria is filled with a juice mixture that is 10% mango and 90% orange juice. A pineapple-mango blend (40% pineapple and 60% mango) is entering the dispenser at a rate of 4 quarts an hour and the well-stirred mixture leaves at a rate of 4 quarts an hour. Model the situation with a differential equation whose solution, $M(t)$, is the amount of mango juice in the container at time t. ($t = 0$ is the time when the pineapple-mango blend starts to enter the dispenser.)

7. Suppose that, in the previous problem, the mixture was leaving at a rate of 5 quarts per hour rather than 4 quarts per hour. Model the new situation.