Second-Order Homogeneous Differential Equations with Constant Coefficients

1. Suppose $f(t)$ and $g(t)$ are both solutions to the differential equation $y''+by'+cy=0$. Is $C_1f(t)+C_2g(t)$ a solution as well?

2. Can you guess solutions of $y''=y$? Try to guess two solutions that are not just multiples of each other.

3. Can you guess solutions of $y''=4y$? Try to guess two solutions that are not just multiples of each other.

4. Solve $y''-y'=6y$.

5. Solve $y'' + 5y' + 4y = 0$ where $y(0) = 1$ and $y'(0) = 2$.

6. Solve $y'' - 4y' + 4y = 0$.

7. Show that, if the characteristic equation $y'' + by' + cy = 0$ has one repeated root r, then $y = te^{rt}$ is a solution to $y'' + by' + cy = 0$.