Second-Order Homogeneous Differential Equations with Constant Coefficients

1. Which of the following differential equations has periodic solutions? What is the period?

   (a) $y'' + 2y' - 3y = 0$.

   (b) $y'' + 2y' + 3y = 0$.

   (c) $y'' + 4y' = 0$.

   (d) $y'' + 4y = 0$.

   (e) $y'' - 4y = 0$.

   Does this agree with your interpretation of the differential equations in terms of vibrating springs?

2. A spring with a mass of 5 kg has a natural length of 6 cm. A 20 N force is required to compress it to a length of 5 cm. If the spring is stretched to a length of 7 cm and released, find the position of the mass at time t. Sketch a graph of the position vs. time.
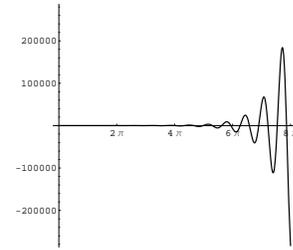
3. Match each differential equation with the graph of its solution. In each case, the differential equation has initial conditions $y(0) = 1$, $y'(0) = 1$.
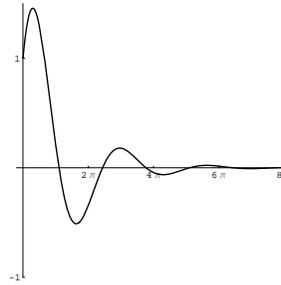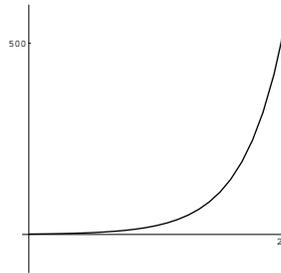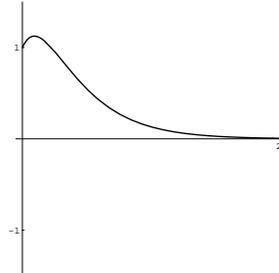
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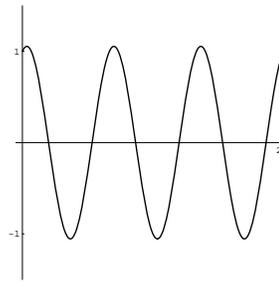
(2)

(3)

(4)

(5)

(a) $y'' + 3y' + 2y = 0$.

(b) $y'' + 9y = 0$.

(c) $y'' - 2y' + y = 0$.

(d) $y'' - y' + 10y = 0$.

(e) $y'' + \frac{1}{2}y' + \frac{5}{8}y = 0$.

4. Solve the differential equation $y'' + y = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.

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