Systems of Differential Equations

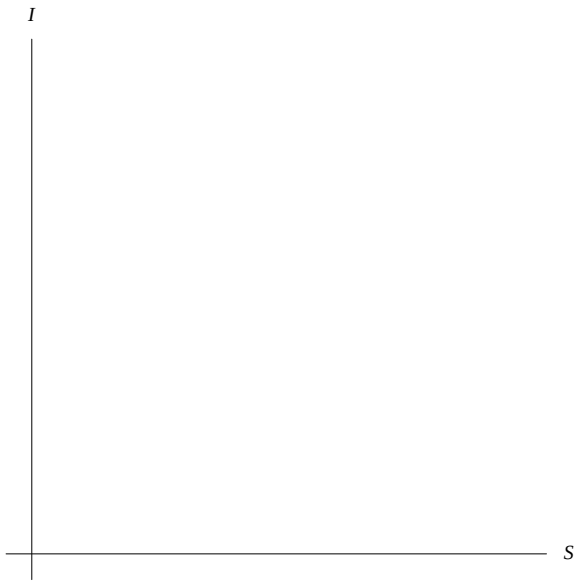
We've used systems of differential equations to model interaction between species. Systems can also be used to model disease epidemics.

Suppose that there is a large population of people, and some of the people have a fatal disease. This disease is infectious, so anybody who doesn't have the disease is susceptible to getting it. Let $I(t)$ be the number of people infected at time t, and let $S(t)$ be the number of people who are susceptible at time t.

1. How could you model this situation with a system of differential equations? You may ignore birth and death, except for death due to the disease, which you should include. (There are many many different answers; when in doubt, opt for simplicity.)

2. Using common sense, find the equilibrium points in this model. (You do not need to use the differential equations you found in #**??**; just think about the situation.)

3. Using common sense, sketch some typical phase trajectories in the phase plane.



4. A reasonable system for the situation described is:

$$\begin{aligned} \frac{dS}{dt} &= -0.001IS \\ \frac{dI}{dt} &= 0.001IS - 0.1I \end{aligned}$$

Sketch the phase portrait for this system. (Be sure to draw the nullclines and equilibrium points.)

5. If the population starts with 50 infected people and 200 susceptible people, what will happen in the long run?