Second Exam for Mathematics 1b

April 10, 2003

Name: ________________________________

Problem	Points	Score
1	18	
2	16	
3	12	
4	21	
5	12	
6	13	
7	8	
Total	100	

You have two hours for this exam. Work carefully and efficiently. Do not spend an inordinate amount of time on any one problem. Think clearly and do well!

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get full credit. If you have work on the back of a page, indicate that on the exam cover.

Please circle your section.

MWF 10	Amanda Alvine	MWF 11	Dylan Thurston	MWF 12	Peter Kronheimer
TTH 10	Robin Gottlieb	TTH 11:30	Alex Popa	TTH 11:30	Robin Gottlieb

Second Examination: April 10, 2003

1. (18 points) Determine whether the following series converge or diverge. Clearly state the test you are using and the evidence backing your conclusion. No credit will be given for a correct answer with no supporting evidence.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{2^n + n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4 + 1}{4^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{\sin k}{k^3 + \sqrt{k}}$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\ln n}$

2. (16 points)

   (a) For what values of x does the series $\displaystyle\sum_{k=3}^{\infty} \frac{(x+3)^k}{k\,4^k}$ converge?

   (If the interval of convergence has endpoints, please determine whether or not the series converges at the endpoints of the interval.)

   (b) You know that $\displaystyle\sum_{n=1}^{\infty} a_n (x-2)^n$ converges for $x = 4$ and diverges for $x = 5.5$. Is there enough information to determine whether it converges for the following values of x. If there is enough information, determine whether the series converges or diverges.

   i. $x = 1$
   ii. $x = 0$
   iii. $x = -4$

   Explain.

3. (12 points)

   (a) Find the Taylor series generated by $x \sin(x^3)$ about $x = 0$.

   (b) What is the radius of convergence of the series in (a)?

   (c) Approximate $\displaystyle\int_0^{0.1} x \sin(x^3)\, dx$ with error less than 10^{-14}. (You may leave your answer as a sum.) Explain your reasoning clearly and completely.

4. (21 points)

   Suppose $\displaystyle\sum_{n=1}^{\infty} a_n$ is a series where $a_n > 0$ for $n > 5$, and $\displaystyle\sum_{n=1}^{\infty} a_n = 0.9$. Let $s_n = a_1 + a_2 + \ldots + a_n$. Which of the following statements must be true? Which must be false? (For some statements it is impossible to tell.) Explain your reasoning. (Full credit will not be awarded without a clear explanation.)

   (a) $\displaystyle\lim_{n\to\infty} a_n = 0.9$

   (b) $\displaystyle\lim_{n\to\infty} s_n = 0.9$

(c) $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(d) If $b_n = a_n + 0.1$ then $\sum_{n=1}^{\infty} b_n$ converges.

(e) $\sum_{n=1}^{\infty} n a_n$ converges.

(f) $\sum_{n=1}^{\infty}(a_n)^{2003}$ converges.

(g) $\sum_{n=1}^{\infty} \frac{a_n}{\sqrt{n}}$ converges.

5. (12 points)

   (a) What is the radius of convergence of the series $\sum_{n=1}^{\infty} n2^n x^{n-1}$?

   (b) What is the Taylor series for $\frac{1}{1-2x}$ about x=0?
   For what values of x does it converge?
   Interval of convergence: ______________________________

   (c) Use your answer to part (b) to determine the sum of the series in part (a) as a function of x, whenever it converges.

6. (13 points)

   (a) What is the Taylor series of $1/x^2$ around $x = -1$?

   (b) For which x does this Taylor series converge? If there are endpoints, test them.
   Interval of convergence: ______________________________

7. (8 points) Which of these polynomials *could* be the second degree Taylor polynomial approximating the function $f(x)$ around the point $x = 2$, if $f(x)$ is the function pictured below? For each answer not chosen, indicate why it was ruled out.

   There was a picture here. (It's not in the file - so consider the following - the graph has a local min at$x = -2$, has a negative $y-$ intercept and is increasing at $x = 0$ with a point of inflection at 0, and is positive, increasing, and concave down at $x = 2$. The answer must work with such a picture.)

   (a) $3(x-2) - 2(x-2)^2$
   (b) $4 - 2(x-2) + (x-2)^2$
   (c) $2 + \frac{1}{2}(x-2) - (x-2)^2$
   (d) $3 + \frac{9}{2}(x-2) + (x-2)^2$