

**Solutions to Review Problems for Differential Equations: Take Two**

1. Graph the differential equation with  $\frac{dw}{dt}$  on the vertical axis and  $w$  on the horizontal axis. Get a parabola that opens upward with zeroes at  $w = 4$  and  $w = 6$ . For  $4 < w < 6$ ,  $\frac{dw}{dt} < 0$ ; thus the solutions  $w(t)$ , must be decreasing for  $4 < w < 6$ . Similarly, for  $w < 4$  or  $w > 6$ , the solutions,  $w(t)$  must be increasing. Answer: (d).

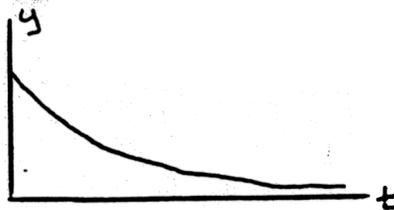
2. The solutions are always increasing. Thus, we must have  $\frac{dP}{dx} \geq 0$ . Answer: (e). Note: (d) and (f) have  $\frac{dP}{dx} < 0$  for negative  $x$ . (b) and (c) have  $\frac{dP}{dx} < 0$  for  $P < 0$ . (a) has  $\frac{dP}{dx} < 0$  for  $P > 0$ .

3. Do a rough phase plane analysis:  $\frac{dx}{dt} = -2$ ,  $\frac{dy}{dt} = 4x$  (Answer: 5)

$$\frac{dx}{dt} = 3y, \quad \frac{dy}{dt} = -3x \text{ (Answer: 9)}$$

$$\frac{dx}{dt} = 10x, \quad \frac{dy}{dt} = 10y \text{ (Answer: 7)}$$

4. Differential equation:  $\frac{dy}{dt} = -ky$ . Solution:  $y = Ce^{-kt}$ .



5. (a) Let  $y(t)$  = bank balance at time  $t$ . Then  $\frac{dy}{dt} = -12,000 + .06y$ . If the initial amount is 200,000, then  $\frac{dy}{dt} = 0$  and  $y$  will remain constant.

(b) Solve by separating variables:

$$\int \frac{dy}{-12,000 + .06y} = \int dt.$$

$$\frac{1}{.06} \ln |.06y - 12,000| = t + c.$$

After a few more steps, you get  $y(t) = 200,000 + Ae^{.06t}$ . If  $y(0) = y_0$ , then  $y = 200,000 + (y_0 - 200,000)e^{.06t}$ . To have  $y(20) = 0$ , we must have  $y(0) \approx 140,000$ .

6. At  $(1, 1)$ ,  $\frac{dx}{dt}$  is positive. This is only the case in (c).

7. (a)  $\frac{dT}{dt} = \frac{1}{29}(10 - T)$

(b)  $\frac{dT}{dt} = \frac{10 - T}{29} \Rightarrow t = -29 \ln(10 - T) + C \Rightarrow t - C = -29 \ln(10 - T) \Rightarrow \ln(10 - T) = -\frac{t}{29} + k \Rightarrow 10 - T = Ke^{-\frac{t}{29}} \Rightarrow T = 10 + Ke^{-\frac{t}{29}}$ . From the initial value  $T(0) = 65$ , we get  $K = 55$ , so  $T = 10 + 55e^{-\frac{t}{29}}$ . Therefore,  $T(5) \approx 56.3$ .

(c)  $\frac{dT}{dt} = 2 + \frac{1}{29}(10 - T)$

$$(2 + \frac{1}{29}(10 - T)) = 0 \text{ for } T = 68$$

(d) Independent of the initial temperature, the temperature will approach  $68^\circ$ .

8. Look at the characteristic eq'n:  $r^2+r+4r=0 \Rightarrow r = -\frac{1}{2} \pm \frac{\sqrt{15}}{2}i$

So  $x(t) = e^{-\frac{1}{2}t} [C_1 \cos \frac{\sqrt{15}}{2}t + C_2 \sin \frac{\sqrt{15}}{2}t] \Rightarrow$  only (a) or (c) is possible.

Only (a) has  $x(0) < 0$  &  $x'(0) < 0$ .

9. Characteristic eq'n:  $r^2-r+1=0$  so  $r = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ ;  $y(t) = e^{t/2} [C_1 \cos \frac{\sqrt{3}}{2}t + C_2 \sin \frac{\sqrt{3}}{2}t]$

$e^{t/2}$  increases with  $t$  so the answer must be (a).

10. h

11. c

12. b

13. a) 5 b) 2

14. a. C only C is always concave up

b. A  $y = -e^t + 2$ ,  $y' < 0$  if  $y < 2$

c. B zero sol'n:  $y = 0$

d. D  $y = e^{-t/2}$