**4.** (a) $y = \cos kt \quad \Rightarrow \quad y' = -k \sin kt \quad \Rightarrow \quad y'' = -k^2 \cos kt$. Substituting these expressions into the differential equation $4y'' = -25y$, we get $4(-k^2 \cos kt) = -25(\cos kt) \quad \Rightarrow \quad (25 - 4k^2) \cos kt = 0 \quad$ [for all $t$] $\quad \Rightarrow$
$25 - 4k^2 = 0 \quad \Rightarrow \quad k^2 = \frac{25}{4} \quad \Rightarrow \quad k = \pm\frac{5}{2}$.

(b) $y = A \sin kt + B \cos kt \quad \Rightarrow \quad y' = Ak \cos kt - Bk \sin kt \quad \Rightarrow \quad y'' = -Ak^2 \sin kt - Bk^2 \cos kt$.
The given differential equation $4y'' = -25y$ is equivalent to $4y'' + 25y = 0$. Thus,

$$\begin{aligned} \text{LHS} = 4y'' + 25y &= 4(-Ak^2 \sin kt - Bk^2 \cos kt) + 25(A \sin kt + B \cos kt) \\ &= -4Ak^2 \sin kt - 4Bk^2 \cos kt + 25A \sin kt + 25B \cos kt \\ &= (25 - 4k^2)A \sin kt + (25 - 4k^2)B \cos kt \\ &= 0 \quad \text{since } k^2 = \tfrac{25}{4}. \end{aligned}$$

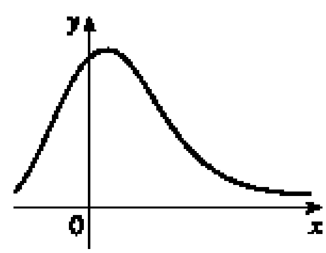
**5.** (a) $y = \sin x \quad \Rightarrow \quad y' = \cos x \quad \Rightarrow \quad y'' = -\sin x$.
LHS $= y'' + y = -\sin x + \sin x = 0 \neq \sin x$, so $y = \sin x$ **is not** a solution of the differential equation.

(b) $y = \cos x \quad \Rightarrow \quad y' = -\sin x \quad \Rightarrow \quad y'' = -\cos x$.
LHS $= y'' + y = -\cos x + \cos x = 0 \neq \sin x$, so $y = \cos x$ **is not** a solution of the differential equation.

(c) $y = \frac{1}{2}x \sin x \quad \Rightarrow \quad y' = \frac{1}{2}(x \cos x + \sin x) \quad \Rightarrow \quad y'' = \frac{1}{2}(-x \sin x + \cos x + \cos x)$.
LHS $= y'' + y = \frac{1}{2}(-x \sin x + 2\cos x) + \frac{1}{2}x \sin x = \cos x \neq \sin x$, so $y = \frac{1}{2}x \sin x$ **is not** a solution of the differential equation.

(d) $y = -\frac{1}{2}x \cos x \quad \Rightarrow \quad y' = -\frac{1}{2}(-x \sin x + \cos x) \quad \Rightarrow \quad y'' = -\frac{1}{2}(-x \cos x - \sin x - \sin x)$.
LHS $= y'' + y = -\frac{1}{2}(-x \cos x - 2\sin x) + \left(-\frac{1}{2}x \cos x\right) = \sin x =$ RHS, so $y = -\frac{1}{2}x \cos x$ **is** a solution of the differential equation.

**12.** The graph for this exercise is shown in the figure at the right.




A. $y' = 1 + xy > 1$ for points in the first quadrant, but we can see that $y' < 0$ for some points in the first quadrant.
B. $y' = -2xy = 0$ when $x = 0$, but we can see that $y' > 0$ for $x = 0$.

Thus, equations A and B are incorrect, so the correct equation is C.

C. $y' = 1 - 2xy$ seems reasonable since:

(1) When $x = 0$, $y'$ could be 1.

(2) When $x < 0$, $y'$ could be greater than 1.

(3) Solving $y' = 1 - 2xy$ for $y$ gives us $y = \dfrac{1 - y'}{2x}$. If $y'$ takes on small negative values, then as $x \to \infty$, $y \to 0^+$, as shown in the figure.

**3.** $y' = y - 2$. The slopes at each point are independent of $x$, so the slopes are the same along each line parallel to the $x$-axis. Thus, III is the direction field for this equation. Note that for $y = 2$, $y' = 0$.

**4.** $y' = x(2-y) = 0$ on the lines $x = 0$ and $y = 2$. Direction field I satisfies these conditions.

**5.** $y' = x + y - 1 = 0$ on the line $y = -x + 1$. Direction field IV satisfies this condition. Notice also that on the line $y = -x$ we have $y' = -1$, which is true in IV.

**6.** $y' = \sin x \sin y = 0$ on the lines $x = 0$ and $y = 0$, and $y' > 0$ for $0 < x < \pi$, $0 < y < \pi$. Direction field II satisfies these conditions.

# *Differential Equations Handout A*

**1.** (a) $\frac{dx}{dt} = -2t$. The general solution is

$$x = -t^2 + C.$$

If $x(0) = 1$, $1 = x(0) = C$. Thus $x = -t^2 + 1$.
If $x(0) = 0$, $0 = x(0) = C$. Thus $x = -t^2$.
If $x(0) = -1$, $-1 = x(0) = C$. Thus $x = -t^2 - 1$.
None of the solutions intersect.

(b) $\frac{dx}{dt} = -2x$. The general solution is

$$x = Ce^{-2t}.$$

If $x(0) = 1$, $1 = x(0) = C$. Thus $x = e^{-2t}$.
If $x(0) = 0$, $0 = x(0) = C$. Thus $x = 0$.
If $x(0) = -1$, $-1 = x(0) = C$. Thus $x = -e^{-2t}$.
None of the solutions intersect.

(c) $\frac{dx}{dt} = t^2$. The general solution is

$$x = \frac{t^3}{3} + C.$$

If $x(0) = 1$, $1 = x(0) = C$. Thus $x = \frac{t^3}{3} + 1$.
If $x(0) = 0$, $0 = x(0) = C$. Thus $x = \frac{t^3}{3}$.
If $x(0) = -1$, $-1 = x(0) = C$. Thus $x = \frac{t^3}{3} - 1$.
None of the solutions intersect.

**4.** $\frac{dC(t)}{dt} = K(L - C(t))$ for some constant K. K is positive because if $L > C(t)$, the solute flows into the cell.

**5.** (a) $\frac{dI}{dt} = k(N - I)I$ where $k$ is positive because people can only become infected, not recover.

(b) The number is increasing as long as $0 < I < N$, which is necessary as I is a subset of the population. Eventually, everyone gets sick.

**7.**

$$\frac{dB}{dt} = B \cdot r - 12000 = 0.0725B - 12000.$$

$B \cdot r$ is the interest accumulated and 12000 is the amount paid.