Name: ___________________ Class:___ Date: ______________

10. Let $u = x^2$. Then $du = 2x\,dx$, so $\int xe^{x^2}\,dx = \int e^u\left(\frac{1}{2}\,du\right) = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$.

14. Let $u = x^2 + 1$. Then $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$, so

$$\int \frac{x}{(x^2+1)^2}\,dx = \int u^{-2}\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\cdot\frac{-1}{u} + C = \frac{-1}{2u} + C = \frac{-1}{2(x^2+1)} + C.$$

30. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$.

32. Let $u = \cos x$. Then $du = -\sin x\,dx$ and $\sin x\,dx = -du$, so

$$\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{-du}{1+u^2} = -\tan^{-1}u + C = -\tan^{-1}(\cos x) + C.$$

33. Let $u = 1 + x^2$. Then $du = 2x\,dx$, so

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx = \tan^{-1}x + \int \frac{\frac{1}{2}du}{u} = \tan^{-1}x + \tfrac{1}{2}\ln|u| + C$$
$$= \tan^{-1}x + \tfrac{1}{2}\ln\left|1+x^2\right| + C = \tan^{-1}x + \tfrac{1}{2}\ln\left(1+x^2\right) + C \quad [\text{since } 1 + x^2 > 0].$$

40. Let $u = 4 + 3x$, so $du = 3\,dx$. When $x = 0$, $u = 4$; when $x = 7$, $u = 25$. Thus,

$$\int_0^7 \sqrt{4+3x}\,dx = \int_4^{25} \sqrt{u}\left(\tfrac{1}{3}\,du\right) = \frac{1}{3}\left[\frac{u^{3/2}}{3/2}\right]_4^{25} = \frac{2}{9}(25^{3/2} - 4^{3/2}) = \frac{2}{9}(125-8) = \frac{234}{9} = 26$$

47. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u^{3/2} + u^{1/2})\,du = \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$.

57. First write the integral as a sum of two integrals:

$I = \int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx = I_1 + I_2 = \int_{-2}^{2} x\sqrt{4-x^2}\,dx + \int_{-2}^{2} 3\sqrt{4-x^2}\,dx$. $I_1 = 0$ by Theorem 6(b), since $f(x) = x\sqrt{4-x^2}$ is an odd function and we are integrating from $x = -2$ to $x = 2$. We interpret I_2 as three times the area of a semicircle with radius 2, so $I = 0 + 3\cdot\frac{1}{2}\left(\pi\cdot 2^2\right) = 6\pi$.

64. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 x f\left(x^2\right)dx = \int_0^9 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$.

9. Let $u = \ln(2x+1)$, $dv = dx \quad \Rightarrow \quad du = \dfrac{2}{2x+1}\,dx$, $v = x$. Then

$$\int \ln(2x+1)\,dx = x\ln(2x+1) - \int \frac{2x}{2x+1}\,dx = x\ln(2x+1) - \int \frac{(2x+1)-1}{2x+1}\,dx$$
$$= x\ln(2x+1) - \int \left(1 - \frac{1}{2x+1}\right)dx = x\ln(2x+1) - x + \tfrac{1}{2}\ln(2x+1) + C$$
$$= \tfrac{1}{2}(2x+1)\,\ln(2x+1) - x + C$$

20. Let $u = \arctan(1/x)$, $dv = dx \quad \Rightarrow \quad du = \dfrac{1}{1+(1/x)^2}\cdot\dfrac{-1}{x^2}\,dx = \dfrac{-dx}{x^2+1}$, $v = x$. Then

$$\int_1^{\sqrt{3}} \arctan(1/x)\,dx = \left[x\arctan\left(\frac{1}{x}\right)\right]_1^{\sqrt{3}} + \int_1^{\sqrt{3}} \frac{x\,dx}{x^2+1} = \sqrt{3}\,\frac{\pi}{6} - 1\cdot\frac{\pi}{4} + \frac{1}{2}\Big[\ln(x^2+1)\Big]_1^{\sqrt{3}}$$
$$= \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2}(\ln 4 - \ln 2) = \frac{\pi\sqrt{3}}{6} - \frac{\pi}{2} + \frac{1}{2}\ln\frac{4}{2} = \frac{\pi\sqrt{3}}{6} - \frac{\pi}{2} + \frac{1}{2}\ln 2$$

23. Let $u = (\ln x)^2$, $dv = dx \quad \Rightarrow \quad du = \dfrac{2}{x}\ln x\,dx$, $v = x$. By Formula 6, $I = \int_1^2 (\ln x)^2\,dx = \left[x(\ln x)^2\right]_1^2 - 2\int_1^2 \ln x\,dx$.

To evaluate the last integral, let $U = \ln x$, $dV = dx \quad \Rightarrow \quad dU = \dfrac{1}{x}\,dx$, $V = x$. Thus,

$$I = [x(\ln x)]_1^2 - 2\left([x\ln x]_1^2 - \int_1^2 dx\right) = [x(\ln x) - 2x\ln x + 2x]_1^2$$
$$= \left(2(\ln 2)^2 - 4\ln 2 + 4\right) - (0 - 0 + 2) = 2(\ln 2)^2 - 4\ln 2 + 2$$

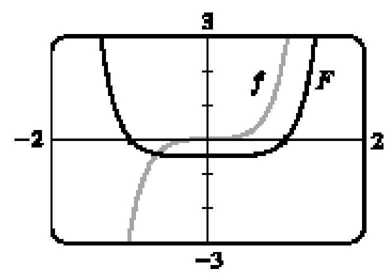
28. Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w\,dw$. Thus, $\int_1^4 e^{\sqrt{x}}\,dx = \int_1^2 e^w 2w\,dw$. Now use parts with $u = 2w$, $dv = e^w\,dw$, $du = 2\,dw$, $v = e^w$ to get $\int_1^2 e^w 2w\,dw = [2we^w]_1^2 - 2\int_1^2 e^w\,dw = 4e^2 - 2e - 2(e^2 - e) = 2e^2$.

32. $\int x^3 e^{x^2}\,dx = \int x^2 \cdot xe^{x^2}\,dx = I$.

Let $u = x^2$, $dv = xe^{x^2}\,dx \quad \Rightarrow \quad du = 2x\,dx$, $v = \frac{1}{2}e^{x^2}$. Then

$I = \frac{1}{2}x^2e^{x^2} - \int xe^{x^2}\,dx = \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C = \frac{1}{2}e^{x^2}(x^2 - 1) + C$.

We see from the graph that this is reasonable, since F has a minimum where f changes from negative to positive.



6. $25 = f(x) = kx = k(0.1)$ [10 cm = 0.1 m], so $k = 250$ N/m and $f(x) = 250x$. Now 5 cm = 0.05 m, so $W = \int_0^{0.05} 250x\,dx = \left[125x^2\right]_0^{0.05} = 125(0.0025) = 0.3125 \approx 0.31$ J.

In Exercises 9–16, n is the number of subintervals of length Δx, and x_i^* is a sample point in the ith subinterval $[x_{i-1}, x_i]$.

10. *Assumptions*:

1. After lifting, the chain is L-shaped, with 4 m of the chain lying along the ground.
2. The chain slides effortlessly and without friction along the ground while its end is lifted.
3. The weight density of the chain is constant throughout its length and therefore equals $(8\text{ kg/m})(9.8\text{ m/s}^2) = 78.4\text{ N/m}$.

The part of the chain x m from the lifted end is raised $6 - x$ m if $0 \le x \le 6$ m, and it is lifted 0 m if $x > 6$ m.

Thus, the work needed is

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} (6 - x_i^*) \cdot 78.4\,\Delta x = \int_0^6 (6 - x) 78.4\,dx = 78.4\left[6x - \tfrac{1}{2}x^2\right]_0^6 = (78.4)(18) = 1411.2\text{ J}$$

In Exercises 9–16, n is the number of subintervals of length Δx, and x_i^* is a sample point in the ith subinterval $[x_{i-1}, x_i]$.

12. The work needed to lift the bucket itself is 4 lb · 80 ft = 320 ft-lb. At time t (in seconds) the bucket is $x_i^* = 2t$ ft above its original 80 ft depth, but it now holds only $(40 - 0.2t)$ lb of water. In terms of distance, the bucket holds $\left[40 - 0.2\left(\tfrac{1}{2}x_i^*\right)\right]$ lb of water when it is x_i^* ft above its original 80 ft depth. Moving this amount of water a distance Δx requires $\left(40 - \tfrac{1}{10}x_i^*\right)\Delta x$ ft-lb of work. Thus, the work needed to lift the water is

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \left(40 - \tfrac{1}{10}x_i^*\right)\Delta x = \int_0^{80} \left(40 - \tfrac{1}{10}x\right)dx = \left[40x - \tfrac{1}{20}x^2\right]_0^{80} = (3200 - 320)\text{ ft-lb}$$

Adding the work of lifting the bucket gives a total of 3200 ft-lb of work.

In Exercises 9–16, n is the number of subintervals of length Δx, and x_i^* is a sample point in the ith subinterval $[x_{i-1}, x_i]$.

16. A horizontal cylindrical slice of water Δx ft thick has a volume of $\pi r^2 h = \pi \cdot 12^2 \cdot \Delta x\text{ ft}^3$ and weighs about $\left(62.5\text{ lb/ft}^3\right)\left(144\pi\,\Delta x\text{ ft}^3\right) = 9000\pi\,\Delta x$ lb. If the slice lies x_i^* ft below the edge of the pool (where $1 \le x_i^* \le 5$), then the work needed to pump it out is about $9000\pi x_i^*\,\Delta x$. Thus,

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 9000\pi x_i^*\,\Delta x = \int_1^5 9000\pi x\,dx = \left[4500\pi x^2\right]_1^5 = 4500\pi(25 - 1) = 108{,}000\pi\text{ ft-lb}$$

18. Let x be depth in feet, so that $0 \le x \le 5$. Then $\Delta W = (62.5)\pi\left(\sqrt{5^2 - x^2}\right)^2 \Delta x \cdot x$ ft-lb and

$$W \approx 62.5\pi \int_0^5 x\left(25 - x^2\right)dx = 62.5\pi\left[\tfrac{25}{2}x^2 - \tfrac{1}{4}x^4\right]_0^5 = 62.5\pi\left(\tfrac{625}{2} - \tfrac{625}{4}\right) = 62.5\pi\left(\tfrac{625}{4}\right) \approx 3.07 \times 10^4\text{ ft-lb}$$