Final copy

Harvard University
Math 1b. Second Semester Calculus
First Exam

Name _____

October 27, 1999

First Exam

- Do not open this exam booklet until you are directed to do so. You will have two hours to complete this exam.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Do not put part of the answer to one problem on the back of the sheet for another problem.
- Don't spend too much time on any one problem. Read them all through first and work on them in the order that allows you to make the most progress.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat. Also, be sure to justify your solutions (unless you are explicitly told otherwise), so we can follow your reasoning.
- Concentrate and do well!

Problem	Points	Grade
1	9	
2	8	
3	12	
4	14	
5	12	
6	15	
7	6	
8	9	
9	12	
bonus	3	
Total	100	

Please circle your section:

MWF 10	Curt McMullen	TTh 10	Yuhan Zha
MWF 10	Pete Clark	TTh 10	Yu-Ru Liu
MWF 11	Tammy Lefcourt	TTh 10	Nina Zipser
MWF 11	Greg Warrington	TTh 11:30	Andy Engelward
MWF 12	Kiril Selverov	TTh 11:30	Alexandru Popa

Problem 1: (9 points total) Determine whether each of the following series converges or diverges. If a series converges, find its sum.

a. (3 points) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{2}{\pi}\right)^k$

b. (3 points) $\displaystyle\sum_{k=2}^{\infty}\left(\frac{2}{\pi}\right)^k$

c. (3 points) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{\pi}{2}\right)^k$

Problem 2: (8 points total) For each of the following infinite series, determine if it converges or diverges. You need to justify your answer to receive full credit (you do not need to specify if convergence is conditional or absolute, just whether the series converges or diverges).

a. (4 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k(k+10)}$

b. (4 points) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)(k+2)}}$

Problem 3: (12 points total) For each of the following infinite series, determine if it converges or diverges. You need to justify your answer to receive full credit (you do not need to specify if convergence is conditional or absolute, just whether the series converges or diverges).

a. (4 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{(k+1)}}{\sqrt{4k}}$

b. (4 points) $\displaystyle\sum_{k=1}^{\infty} \frac{k^4+1}{k!}$

c. (4 points) $\displaystyle\sum_{k=1}^{\infty} \frac{|\sin k|}{(k+3)^{\frac{3}{2}}}$

Problem 4: (14 points total) Find the interval of convergence and radius of convergence for each of the following power series (be sure to check convergence at endpoints).

a. (7 points) $\displaystyle\sum_{k=0}^{\infty} \frac{(x-5)^k}{\ln(k+2)}$

b. (7 points) $\displaystyle\sum_{k=0}^{\infty} \frac{(-x)^k 2^k}{k!}$

Problem 5: (12 points total)

a. (7 points total) Write out the first four non-zero terms of the Taylor series about $x = 0$ (*i.e.* the Maclaurin series) for the function $f(x) = \dfrac{x}{1+x}$.

b. (5 points) Write out the first four non-zero terms of the Taylor series about $x = 0$ for $g(x) = \int f(x)dx$, assuming that $g(0) = 2$.

Problem 6: (15 points total) Consider the function $f(x) = \cos(x^3)$

a. (6 points) Write out the first four non-zero terms of the Taylor series about $x = 0$ (*i.e.* the Maclaurin series) for $f(x)$.

b. (3 points) Does this Taylor series converge for $x = 10$? Justify your answer.

c. (6 points) Using the identity $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$, write out the first four non-zero terms of the Taylor series about $x = 0$ for $\cos^2 x$.

Problem 7: (6 points total) Consider the following infinite series

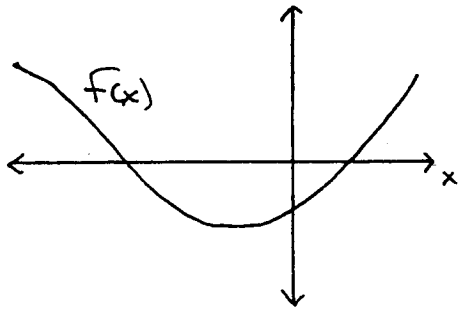
$$1 + e^{-2x} + e^{-4x} + e^{-6x} + e^{-8x} + \ldots$$

a. **(3 points)** For what values of x does this series converge?

b. **(3 points)** For the values of x for which the series converges, find the sum as a function of x.

Problem 8: (9 points total)

Let $f(x)$ be the function whose graph is given in the figure shown below.



Suppose the Taylor series about $x = 0$ for $f(x)$ is:

$$a_0 + a_1 x + a_2 x^2 + \ldots,$$

then circle the correct statement on each of the following three lines:

(1)	$a_0 < 0$	$a_0 = 0$	$a_0 > 0$
(2)	$a_1 < 0$	$a_1 = 0$	$a_1 > 0$
(3)	$a_2 < 0$	$a_2 = 0$	$a_2 > 0$

Problem 9: (12 points total)

a. (6 points) Write out the first four non-zero terms of the Taylor series about $x = \pi/2$ for $f(x) = \sin x$.

b. (6 points) Write out the first four non-zero terms of the Taylor series about $x = \pi/2$ for $\cos x$ by differentiating the above series.