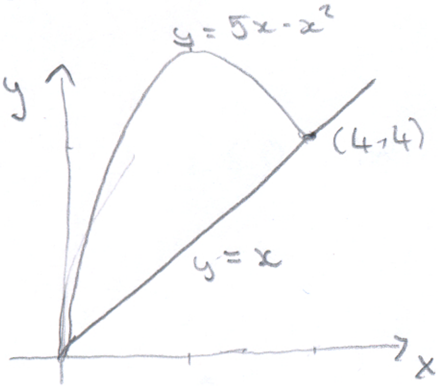
Problem Set # 1
Solutions
Assigned Tues June 29

Section 7.1

(1)



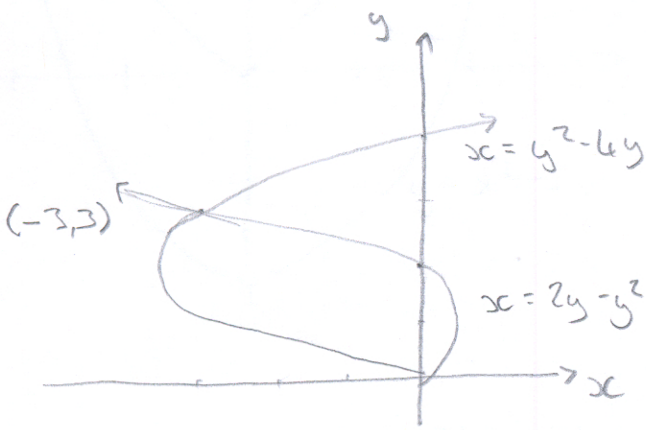
$$\int_0^4 (5x - x^2) - x \, dx$$

$$= \frac{5x^2}{2} - \frac{x^3}{3} - \frac{x^2}{2} \Big|_0^4$$

$$= 40 - \frac{64}{3} - 8$$

$$= \frac{32}{3}$$

(4)



$$\int_0^3 (2y - y^2) - (y^2 - 4y) \, dy$$

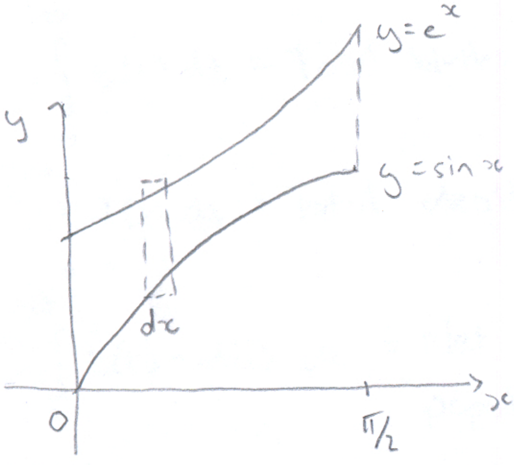
$$= \int_0^3 -2y^2 + 6y \, dy$$

$$= -\frac{2y^3}{3} + \frac{6y^2}{2} \Big|_0^3$$

$$= -18 + 27$$

$$= 9$$

(6)



$$\text{Area} = \int_0^{\frac{\pi}{2}} e^x - \sin x \, dx$$

$$= e^x + \cos x \Big|_0^{\pi/2}$$

$$= e^{\pi/2} + 0 - 1 - 1$$

$$= e^{\pi/2} - 2$$

(16) $y = |x|$

$y = x^2 - 2$

When $x \geq 0$: Intersection when $x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

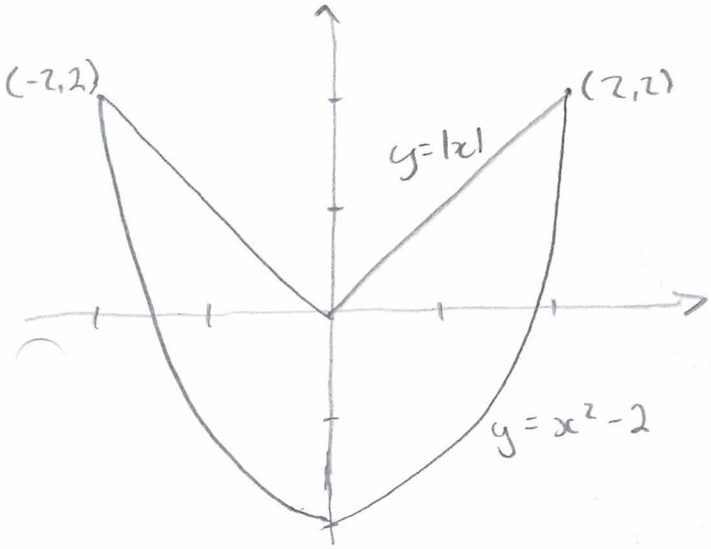
so $\boxed{x = 2}$ $y = 2$

$x \leq 0$: Intersection when $x^2 - 2 = -x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

so $\boxed{x = -2}$ $y = 2$



Use symmetry

$$\text{Area} = 2\int_0^2 x - (x^2 - 2)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_0^2$$

$$= 2\left(2 - \frac{8}{3} + 4\right) = \frac{10}{3} \cdot 2 = \frac{20}{3}$$

(27) $b(t) = 2200\, e^{0.024t}$ = # babies born per year

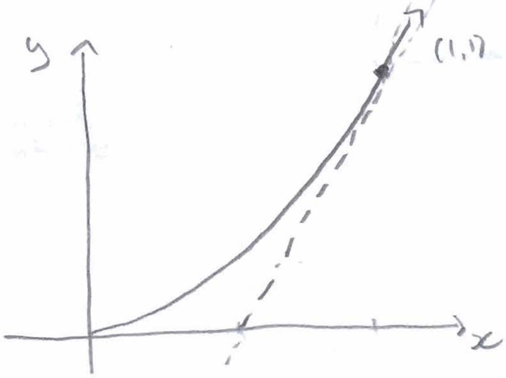
$d(t) = 1460\, e^{0.018t}$ = # people die per year

$\int_0^{10} b(t)\,dt$ = Total births in 10 years

$\int_0^{10} d(t)\,dt$ = Total deaths in 10 years

$\int_0^{10} b(t) - d(t)\,dt$ = Net # people introduced into population each year

$$= \frac{2200 e^{0.024t}}{0.024} - \frac{1460 e^{0.018t}}{0.018}\Bigg|_0^{10} = 8868$$

32) 

$y = x^2$

Slope $= \frac{dy}{dx} = 2x$

At $x=1$, slope $= 2(1) = 2$

$y = mx + c$

We have 1 point $(1,1)$ and the slope, $m = 2$

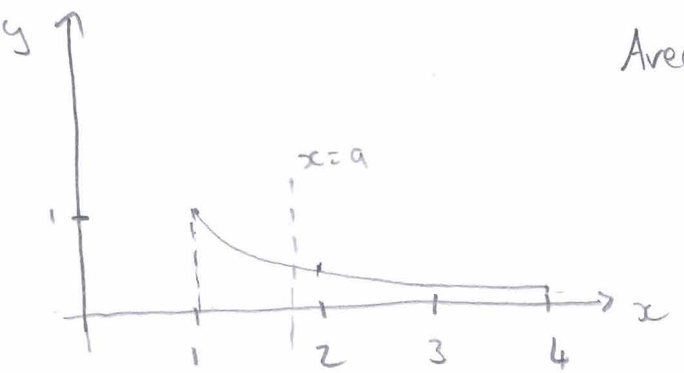
So $1 = 2(1) + c \Rightarrow c = -1$

Integrate in y-direction between $y = 2x - 1$ and $y = x^2$

Rearranging : $x = \frac{y}{2} + \frac{1}{2}$ and $x = \sqrt{y}$

$$\int_0^1 \frac{y}{2} + \frac{1}{2} - \sqrt{y}\, dy = \frac{y^2}{4} + \frac{y}{2} - \frac{2}{3} y^{3/2} \Big|_0^1$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$$

34) a) 

Area under $y = \frac{1}{x^2}$ between $1 \le x \le 4$

$$\int_1^4 \frac{1}{x^2}\, dx = -\frac{1}{x}\Big|_1^4 = -\frac{1}{4} + 1 = \frac{3}{4}$$

We want to find a value of "a" such that the area between $x = 1$, $y = \frac{1}{x^2}$, the x-axis, and $x = a$ is $\frac{1}{2}\left(\frac{3}{4}\right)$ or $\frac{3}{8}$

$$\int_1^a \frac{1}{x^2}\, dx = -\frac{1}{x}\Big|_1^a = -\frac{1}{a} + 1 = \frac{3}{8}$$

$$\frac{1}{a} = \frac{5}{8}$$

$$\Rightarrow a = \frac{8}{5}$$

b) $\frac{16}{13}$

Integration Handout B

(1) Intersection

$-x^2+2 = x$

$x^2+x-2=0$

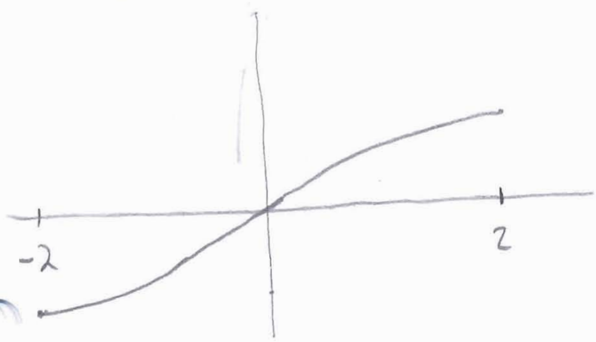
$(x+2)(x-1)=0$

$x=-2$

$x=1$

$$\int_{-2}^{1} -x^2+2-x \, dx$$

(3) a) $y = \arctan x$



By symmetry we see

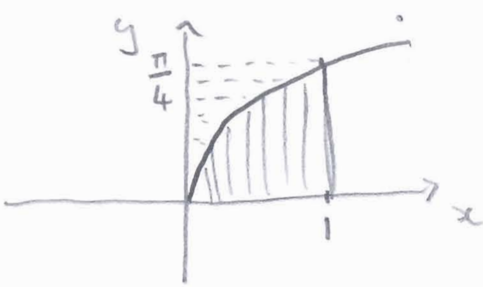
$$\int_{-2}^{2} \arctan x \, dx = 0$$

(Since $\arctan x$ is an odd function, that is, $f(x) = -f(-x)$, the positive area cancels the negative area)

b) $y = \arctan x$ or, equivalently, $x = \tan y$

When $x=0$ $\tan y = 0$ so $y=0$

$x=1$ $\tan y = 1$ so $y = \frac{\pi}{4}$

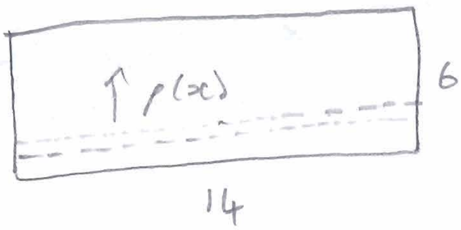


$$\int_0^1 \arctan x \, dx = 1 \times \frac{\pi}{4} - \int_0^{\pi/4} \tan y \, dy$$

$$= \frac{\pi}{4} - \left[\ln\left(\frac{1}{\cos y}\right)\right]_0^{\pi/4}$$

$$= \frac{\pi}{4} - \ln\left|\frac{1}{\sqrt{2}}\right|$$

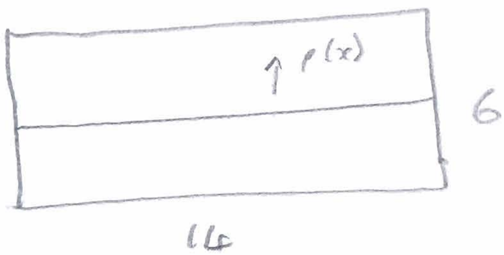
6)



a) Cut the platter into horizontal strips and multiply the strips area by a sample density inside that strip

b) Amount of Cobalt $= 14 \int_0^6 p(x)\,dx$

7)

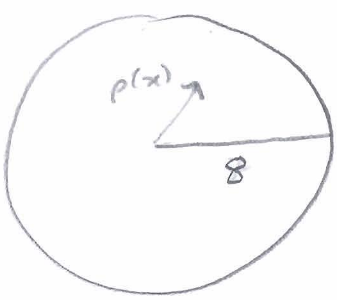


a) Cut into n horizontal strips

b) $2\left(\int_0^3 14\, p(x)\,dx \right)$

8) a) Cut into circular strips

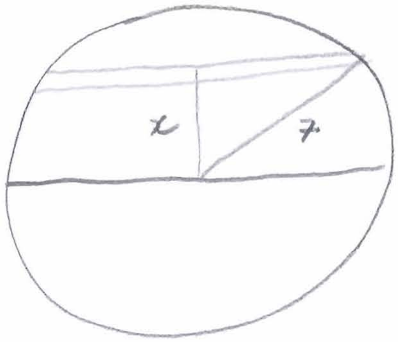
b)



Area 1 strip $= 2\pi r\,dr$

$$\int_0^8 2\pi r\, p(r)\,dr$$

9)



$$\text{Amnt Cobalt} = 2\int_0^8 \rho(x) \times \text{length strip} \times dx$$

$$\text{Length of strip} = 2\sqrt{8^2 - x^2}$$

$$\text{Amt. Cobalt} = 2\int_0^8 \rho(x)\, 2\sqrt{8^2 - x^2}\, dx$$

$$= 4\int_0^8 \rho(x)\sqrt{8^2 - x^2}\, dx$$

10) $$\#\text{ holes} = \int_0^{10} 2\pi \cancel{x}\, \rho(x)\, dx$$

$$= \int_0^{10} 2\pi x \cdot \frac{1010}{\pi(x^2+1)}\, dx$$

$$= \int_0^{10} \frac{2020x}{x^2+1}\, dx \sim 4661 \text{ holes}$$