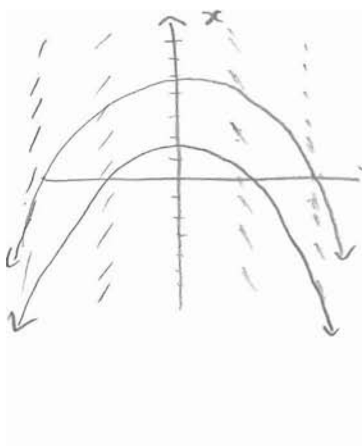
Problem Set #10 Diff Eqns Handout

(1) (a)



$\frac{dx}{dt} = -2t$ Try $x(t) = -t^2 + C$

$\frac{dx}{dt} = \frac{d}{dt}(-t^2) = -2t$ ✓

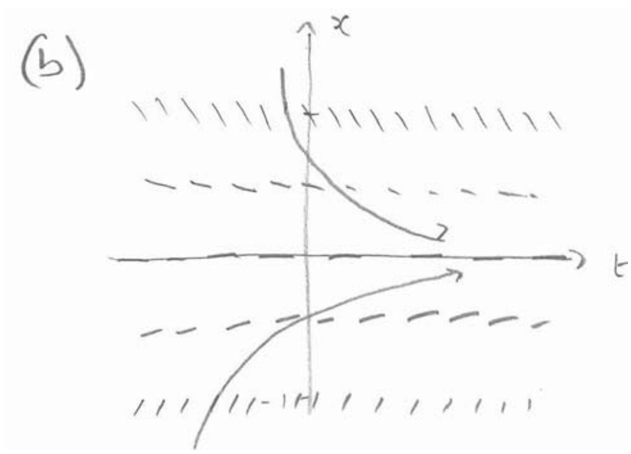
$x(0) = C = 1 \quad : \quad x(t) = -t^2 + 1$

$x(0) = C = 0 \quad : \quad x(t) = -t^2$

$x(0) = C = -1 \quad : \quad x(t) = -t^2 - 1$

No intersections

(b)



$\frac{dx}{dt} = -2x$ Try $x(t) = Ce^{-2t}$

$\frac{dx}{dt} = C(-2e^{-2t})$

$= -2x$ ✓

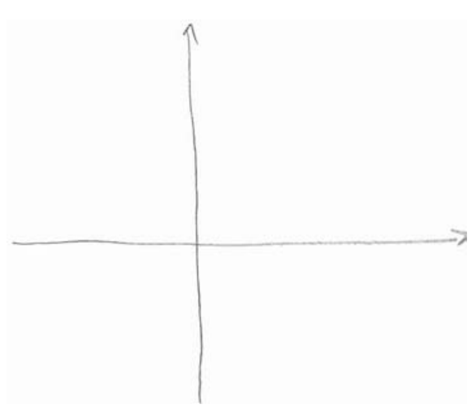
$x(0) = C = 1 \quad x(t) = e^{-2t}$

$x(0) = C = 0 \quad x(t) = 0$

$x(0) = C = -1 \quad x(t) = -e^{-2t}$

No intersections

(c)



$\frac{dx}{dt} = t^2$ Try $x(t) = \frac{1}{3}t^3 + C$

$\frac{dx}{dt} = t^2$ ✓

$x(0) = C = 1 \quad x(t) = \frac{1}{3}t^3 + 1$

$x(0) = C = 0 \quad x(t) = \frac{1}{3}t^3$

$x(0) = C = -1 \quad x(t) = \frac{1}{3}t^3 - 1$

No intersections

(2) (d) $y = Ce^t - 1$

$$\frac{dy}{dt} = \underline{\underline{Ce^t}} \quad \Rightarrow \quad y + 1 = Ce^t - 1 + 1 = \underline{\underline{Ce^t}}$$

(3) $y'' + 9y = 0$

(e) $y = 5\cos 3t$

$y'' = -9.5\cos 3t$

$y'' + 9y = -45\cos 3t + 45\cos 3t = 0$

(4) $C(t)$ = concentration of solute in cell

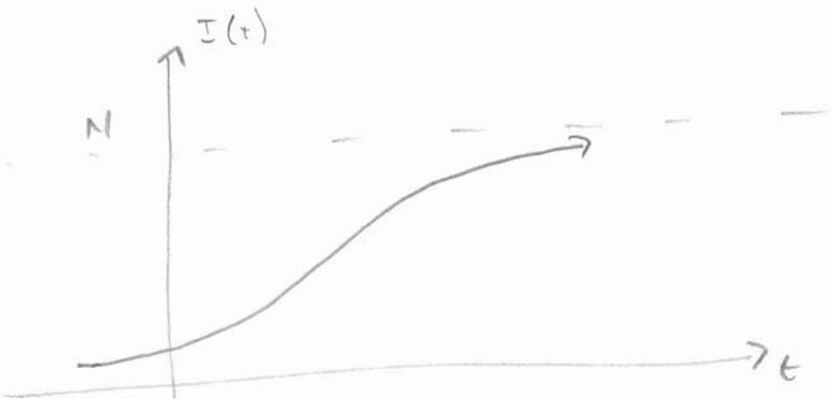
$$\frac{dC(t)}{dt} \propto L - C(t)$$

$$\frac{dC}{dt} = k(L - C)$$

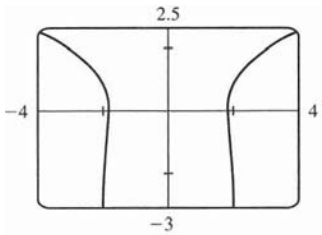
(5) $I(t)$ = # infected at time t

$$\frac{dI}{dt} \propto I(N - I)$$

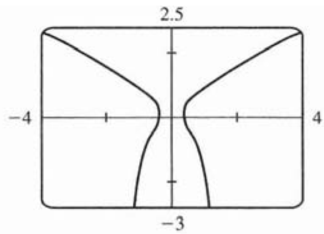
$$\frac{dI}{dt} = kI(N - I) \quad \text{where } k > 0$$



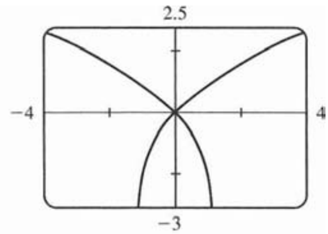
or `Plot[Evaluate[···]]` in Mathematica to plot the solution curves for various values of C.



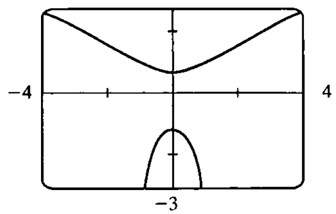
$C = -4$



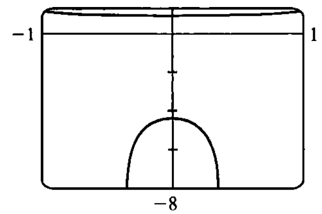
$C = -1.4$



$C = -4/3$

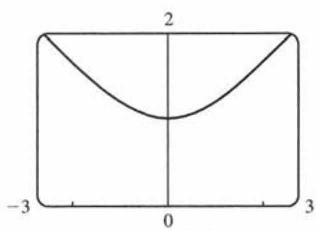


$C = -1$

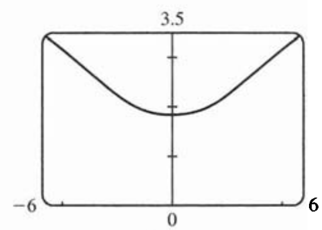


$C = -0.4$

It seems that the transitional values of C are $-\frac{4}{3}$ and $-\frac{1}{3}$. For $C < -\frac{4}{3}$, the graph consists of left and right branches. At $C = -\frac{4}{3}$, the two branches become connected at the origin, and as C increases, the graph splits into top and bottom branches. At $C = -\frac{1}{3}$, the bottom half disappears. As C increases further, the graph moves upward, but doesn't change shape much.



$C = -1/3$



$C = 5$

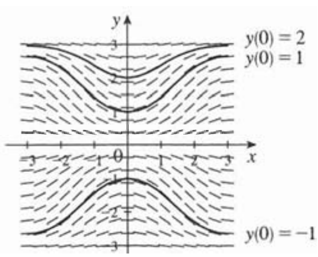
21. $y' = 2 - y$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, III is the direction field for this equation. Note that for $y = 2$, $y' = 0$.

22. $y' = x(2 - y) = 0$ on the lines $x = 0$ and $y = 2$. Direction field I satisfies these conditions.

23. $y' = x + y - 1 = 0$ on the line $y = -x + 1$. Direction field IV satisfies this condition. Notice also that on the line $y = -x$ we have $y' = -1$, which is true in IV.

24. $y' = \sin x \, \sin y = 0$ on the lines $x = 0$ and $y = 0$, and $y' > 0$ for $0 < x < \pi$, $0 < y < \pi$. Direction field II satisfies these conditions.

25. (a) $y(0) = 1$
(b) $y(0) = 2$
(c) $y(0) = -1$



26. (a) $y(0) = -1$
(b) $y(0) = 0$
(c) $y(0) = 1$