Problem Set # 12 # 12, 13, 14, 15 (a = 3/2), 16

(12) If increasing $\Rightarrow f'(x) > 0$

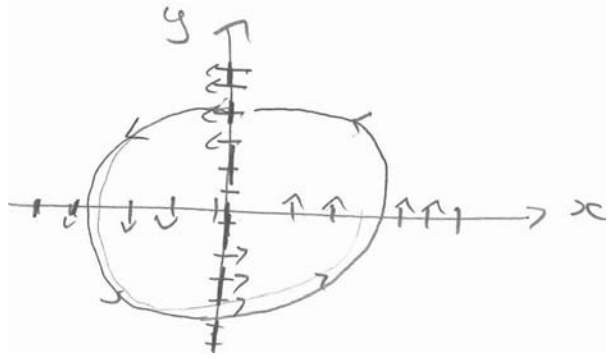
If concave up $\Rightarrow f''(x) > 0$

If both are true then $y'' + y' > 0$

so the equation $y'' + y' = -x^2$ cannot hold as $-x^2$ is -ve

(13) a) $\frac{dx}{dt} = -y$

$\frac{dy}{dt} = x$



b) $x(t) = 4\cos t$ $\quad y(t) = 4\sin t$

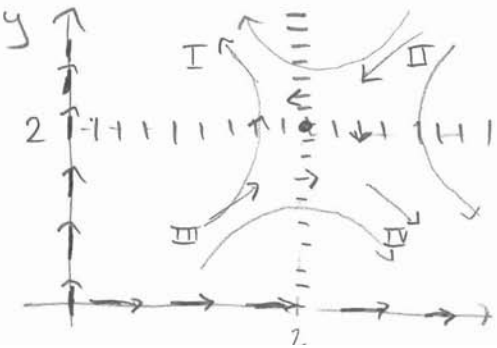
$\frac{dx}{dt} = -4\sin t = -y$ ✓ $\quad \frac{dy}{dt} = 4\cos t = x$ ✓

$x^2 + y^2 = 4^2(\cos^2 t + \sin^2 t) = 16$ ✓

(14) $\frac{dx}{dt} = .1x - .05xy = .1x(1 - .5y)$ a) competitive

$\frac{dy}{dt} = .1y - .05xy = .1y(1 - .5x)$ b) $x = 2,\ y = 2$

c)



	I	II	III	IV
$\frac{dy}{dt}$	+	−	+	−
$\frac{dx}{dt}$	−	−	+	+
$\frac{dy}{dx}$	−	+	+	−

(e) $x = 0 \Rightarrow y(t)$ increases without bound

$y = 0 \Rightarrow x(t)$ " " "

(f) Previous page

(g) $x(0) = 2 \quad y(0) = 1.8 \Rightarrow y \to 0, \; x \to \infty$

$x(0) = 2 \quad y(0) = 2.3 \Rightarrow x \to 0, \; y \to \infty$

$y(0) = 2.2 \quad y(0) = 2 \Rightarrow x \to \infty, \; y \to 0$

(h) Yes!

(15) (a) y is predator : coeff of xy is +ve

x is prey : " " " is -ve

(b) $x > 0, y = 0$: expect x to increase

$y > 0, x = 0$: " y " decrease

(c) Solve $ax - bx^2 - cxy = 0$

$-dy + exy = 0$

(d) a = birth rate of x

b = competition constant for resources of x

c = fraction of interactions between x, y that result in x death

d = death rate of y

e = rate at which predator/prey interactions help growth of y

Problem Set # 12 Scpp 31.5 # 1, 13

(1)

	Interaction Type	Absence of x	Absence of y
a)	Competitive	y reaches a +ve equilibrium	x reaches a +ve equilibrium
b)	Predator / Prey	y dies out	x thrives
c)	Symbiotic	y reaches a +ve equilibrium	x reaches a +ve equilibrium

(13)

a → V

b → IX

c → VIII