Problem Set #3 Solutions

Section 6.1

(8) $\int x^2 \cos mx \, dx$

$u = x^2 \quad dv = \cos mx \, dx$

$du = 2x\,dx \quad v = \frac{1}{m} \sin mx$

$$\int x^2 \cos mx \, dx = \frac{x^2}{m} \sin mx - \int \frac{\sin mx}{m} \cdot 2x \, dx$$

$$= \frac{x^2}{m} \sin mx - \left[\frac{2x(-\cos mx)}{m^2} - \int -\frac{\cos mx}{m^2} \cdot 2 dx \right]$$

$$= \frac{x^2}{m} \sin mx + \frac{2x \cos mx}{m^2} + \frac{2 \sin mx}{m^3} + C$$

(10) $\int p^5 \ln p \, dp = \frac{p^6}{6} \ln p - \int \frac{p^6}{6} \cdot \frac{1}{p} dp = \frac{p^6 \ln p}{6} - \frac{p^6}{36} + C$

$u = \ln p$

$du = \frac{1}{p} dp$

(13) $\int e^{2\theta} \sin 3\theta \, d\theta = \frac{e^{2\theta}}{2} \sin 3\theta - \int \frac{e^{2\theta}}{2} \cdot 3\cos 3\theta \, d\theta$

$u = \sin 3\theta$

$du = 3\cos 3\theta \, d\theta$

$$= \frac{e^{2\theta} \sin 3\theta}{2} - \left[\frac{e^{2\theta}}{4} \cdot 3\cos 3\theta - \int \frac{3e^{2\theta}}{4} \left(\frac{-\sin 3\theta}{\cancel{3}} \right) d\theta \right]$$

$$= \frac{e^{2\theta} \sin 3\theta}{2} - \frac{3}{4} e^{2\theta} \cos 3\theta - \frac{1}{4} \int e^{2\theta} \sin 3\theta \, d\theta$$

$$\Rightarrow \int e^{2\theta} \sin 3\theta \, d\theta = \frac{4}{5} \left[\frac{e^{2\theta} \sin 3\theta}{2} - \frac{3}{4} e^{2\theta} \cos 3\theta \right] + C$$

(17) $u = \ln x \qquad du = \frac{1}{x} dx$

$$\int_1^2 \frac{\ln x}{x^2} dx = -x^{-1} \ln x - \int_1^2 \frac{-1}{x} \cdot \frac{1}{x} dx$$

$$= -\frac{\ln x}{x} \Big|_1^2 + \frac{-1}{x} \Big|_1^2 = -\frac{\ln 2}{2} + \frac{\ln 1}{1} - \left(-\frac{1}{2} + 1 \right)$$

$$= -\frac{\ln 2}{2} - \frac{1}{2}$$

(20) $\displaystyle\int_1^{\sqrt{3}} \arctan(1/x)\,dx = x \arctan\frac{1}{x}\Big|_1^{\sqrt{3}} - \int \frac{-x}{x^2+1}\,dx$

$u = \arctan\left(\frac{1}{x}\right)$

$du = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{-1}{x^2}\,dx = \frac{-1}{x^2+1}\,dx$

$= x\arctan\frac{1}{x} + \frac{1}{2}\ln|x^2+1| \Big|_1^{\sqrt{3}}$

$= \sqrt{3}\arctan\frac{1}{\sqrt{3}} - \arctan 1 + \frac{1}{2}\ln 4 - \frac{1}{2}\ln 2$

$= \sqrt{3}\cdot\frac{\pi}{6} - \frac{\pi}{2} + \frac{1}{2}\ln 2$

(22) $\displaystyle\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr = \frac{1}{2}\int_0^1 r^2 \cdot \frac{2r}{\sqrt{4+r^2}}\,dr$

$u = r^2$

$du = 2r\,dr$

$dv = \frac{2r}{\sqrt{4+r^2}}\,dr$

$v = 2\sqrt{4+r^2}$

$= \frac{1}{2}\left[r^2 2\sqrt{4+r^2} - \int_0^1 2.2r\sqrt{4+r^2}\,dr \right]$

$= \frac{1}{2}\left[2r^2\sqrt{4+r^2} - 2.\frac{2}{3}(4+r^2)^{3/2} \right]_0^1$

$= \frac{1}{2}\left[2\sqrt{5} - \frac{4}{3}(5)^{3/2} + \frac{4}{3}.8 \right]$

$= \sqrt{5} - \frac{10}{3}\sqrt{5} + \frac{16}{3} = \frac{16}{3} - \frac{7}{3}\sqrt{5}$

(25) Let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\Rightarrow 2u\,du = dx$

$\int \sin\sqrt{x}\,dx = \int 2u \sin u\,du$

$= -2u\cos u - \int -2\cos u\,du$

$= -2u\cos u + 2\sin u + C$

(34) $\displaystyle\int x^n e^x\,dx = \underset{u\ v}{x^n e^x} - \int \underset{v}{e^x} . \underset{du}{nx^{n-1}}\,dx$

$u = x^n$

$du = nx^{n-1}$

$dv = e^x dx$

$v = e^x$

$= x^n e^x - n\int x^{n-1} e^x\,dx$

$\boxed{\#6.2}$

(6) $\int \sin^3(mx)\,dx = \int \sin(mx)(1-\cos^2 mx)\,dx$

$$= \underbrace{\int \sin(mx)\,dx}_{(I)} - \underbrace{\int \sin(mx)\cos^2(mx)\,dx}_{(II)}$$

$(I) = \frac{-\cos(mx)}{m}$

(II) Let $\cos mx = u$ $\quad \int mu\,du = \frac{mu^2}{2} = \frac{m\cos^2 mx}{2}$

$\frac{-\sin mx}{m} = du$

$(I)+(II) = \frac{-\cos mx}{m} + \frac{m}{2}\cos^2 mx + C$

$\boxed{\#\ 6.3}$

(1) a) $\frac{A}{x+3} + \frac{B}{3x+1}$

b) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

(2) a) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

b) $\frac{A}{x} + \frac{Bx+C}{x^2+1}$

(12) $\frac{x-1}{(x^2+3x+2)} = \frac{x-1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$\Rightarrow x-1 = Ax+A+Bx+2B$

coeff $x: \ 1 = A+B$ $\quad B=-2$

$C: \ -1 = A+2B$ $\quad A=3$

$\int_0^1 \frac{3}{x+2} - \frac{2}{x+1}\,dx$

$= 3\ln(x+2) - 2\ln(x+1)\Big|_0^1$

$= 3\ln 3 - 2\ln 2 - 3\ln 2$

$= 3\ln 3 - 5\ln 2$

(18) $$\frac{x^2+2x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$x^2+2x-1 = A(x^2-1) + Bx(x+1) + Cx(x-1)$$

coeff x^2: $1 = A+B+C$ | $A=1$ | $B=1$

x : $2 = B - C$ | $B+C=0$ | $C=-1$

c : $-1 = -A$ | $B-C=2$ |

$$\int \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x+1}\,dx = \ln x + \ln|x-1| - \ln|x+1| + C$$

(31) $$\frac{1}{x^2(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

$$1 = Ax(x^2-1) + B(x^2-1) + Cx^2(x+1) + Dx^2(x-1)$$
$$= Ax^3 - Ax + Bx^2 - B + Cx^3 + Cx^2 + Dx^3 - Dx^2$$

Coeff x^3: $0 = A+C+D$ | $B=-1$ |

x^2: $0 = B+C-D$ | $A=0$ |

x : $0 = -A$ | $0=C+D$ | $C=1/2$

c : $1 = -B$ | $1=C-D$ | $D=-1/2$

$$\int \frac{-1}{x^2} + \frac{1/2}{x-1} - \frac{1/2}{x+1}\,dx = \frac{1}{x} + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$

(35) $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2u\,du = dx$

$$\int \frac{\sqrt{x}}{x-4}\,dx = \int \frac{u}{u^2-4}\,2u\,du$$
$$= 2\int \frac{u^2-4+4}{u^2-4}\,du = 2\int 1 + \frac{4}{u^2-4}\,du$$
$$= 2 + \ln\left(\frac{25}{9}\right)$$

(39) Let $u = e^{x}$

$du = e^{x}\,dx$

$\frac{du}{u} = dx$

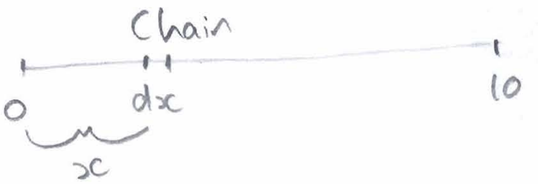
$$\int \frac{e^{2x}}{e^{2x}+3e^{x}+2}\,dx = \int \frac{u^{x}}{u^{2}+3u+2}\cdot\frac{du}{u}$$

$$= \int \frac{u}{(u+2)(u+1)}\,du$$

Using Partial Fractions $\quad = \ln\left[\frac{(e^{x}+2)^{2}}{e^{x}+1}\right] + C$

Section # 7.5

(10)



Chain's mass per unit length $= \frac{80}{10} = 8\,kg\cdot m^{-1}$

weight per unit length $= 8g$

Work on dx $=$ height $\times\ 8g\,dx$

$= (6-x)\,8g\,dx$

Total Work $= \int_0^6 (6-x)\,8g\,dx = 48x - \frac{8x^2}{2}\Big|_0^6$

$= 288 - 144 = 144\ J$

Integration Handout

(20) a) $\int \sin^2 x\,(1-\sin^2 x)\cos x\,dx$

$= \int \cos x \sin^2 x\,dx - \int \sin^4 x \cos x\,dx$

$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

b) $\int (1-\cos^2 x)^2 \cos^2 x \sin x\,dx = -\int (1-u^2)^2 u^2\,du = -\int u^2 - 2u^4 + u^6\,du$

$u = \cos x$

$= -\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$

(21) $\int \sin^2\theta \, d\theta = \int \frac{1}{2} - \frac{\cos 2\theta}{2} \, d\theta$

$= \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C$

(22) $\int_0^3 \sqrt{9 - x^2} \, dx = \int_0^{\pi/2} 3\cos t \,.\, 3\cos t \, dt = 9 \int_0^{\pi/2} \cos^2 t \, dt$

Let $x = 3\sin t$
$dx = 3\cos t \, dt$

$x = 3 : \sin t = 1 \Rightarrow t = \pi/2$

$x = 0 : \sin t = 0 \Rightarrow t = 0$

$= 9 \int_0^{\pi/2} \frac{1}{2} + \frac{\cos 2t}{2} \, dt$

$= 9 \left[\frac{1}{2}t + \frac{\sin 2t}{4} \right]_0^{\pi/2}$

$= 9 \left[\frac{\pi}{4} + 0 - 0 \right]$

$= \frac{9\pi}{4}$

Area of Quater Cirle