Problem Set #6 Solutions

(19) $$\frac{2}{(n-1)(n+1)} = \frac{A}{n-1} + \frac{B}{n+1}$$

$$2 = A(n+1) + B(n-1)$$

$$A+B=0 \quad | \quad A=1$$
$$A-B=2 \quad | \quad B=-1$$

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \frac{1}{n-1} + \frac{1}{n+1} = \frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} \cdots$$

$$= 1 + \frac{1}{2} = \frac{3}{2} \quad \text{Converges}$$

(30) $$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) = \sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$$

$$= \sum_{n=1}^{\infty} \ln(n+1) - \ln(n)$$

$$= (\ln 2 - \ln 1) + (\ln 3 - \ln 2) + (\ln 4 - \ln 3) + \cdots + (\ln(n+1) - \ln n)$$

$$= \lim_{n\to\infty} \ln|n+1| \to \infty$$

This thus diverges

Series Handout (Pset 6 Solutions)

(7) c) $f(x) = \sin \pi x$

$\lim_{x \to \infty} \sin \pi x$ does not exist — it oscillates between -1 and 1

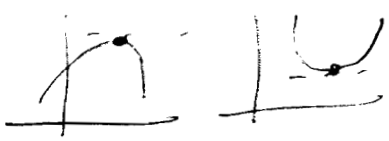
$\lim_{n \to \infty} \sin n\pi = 0$

(10) a) $P_1(x) = 4x$ because this is linear portion of given polynomial

$P_2(x) = -4x^2 + 4x$

$P_3(x) = x^3 - 4x^2 + 4x$

b) A critical point at $x = 2$ means either a maximum or a minimum.



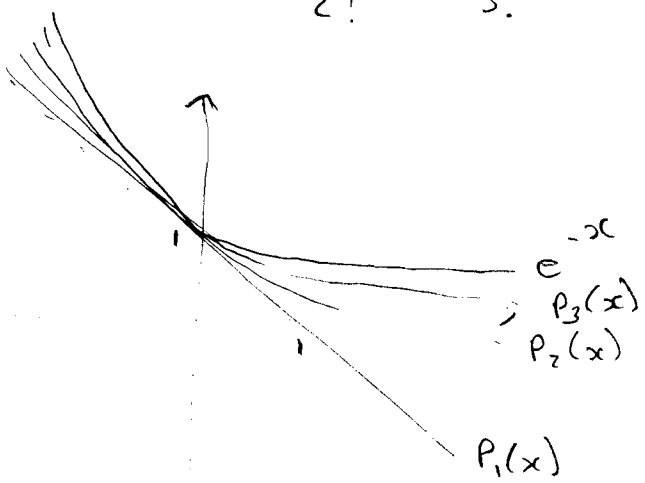
The tangent to a max/min is zero, so the linear term must have zero coefficient

c) $a_4 = a_5 = 0$ since $P_3(x) = f(x)$

...oblems from Chpt 30 Supplement (Sol)

① a) $f(x) = e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$

b)



c) $P_1(.1) = .9$ $\quad P_1(.3) = .7$

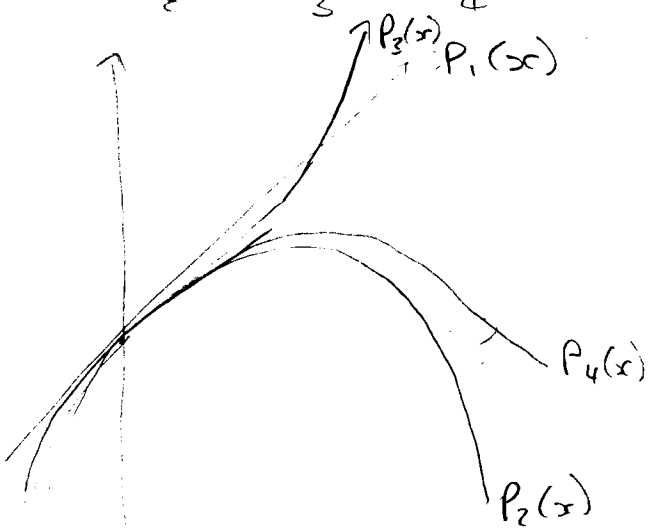
$P_2(.1) = .905$ $\quad P_2(.3) = .745$

$P_3(.1) = .90423$ $\quad P_3(.3) = .7405$

$P_4(.1) = .904837$ $\quad P_4(.3) = .740837$

① a) $f(x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

b)



c) $P_1(.1) = .1$ $\quad P_1(.3) = .3$

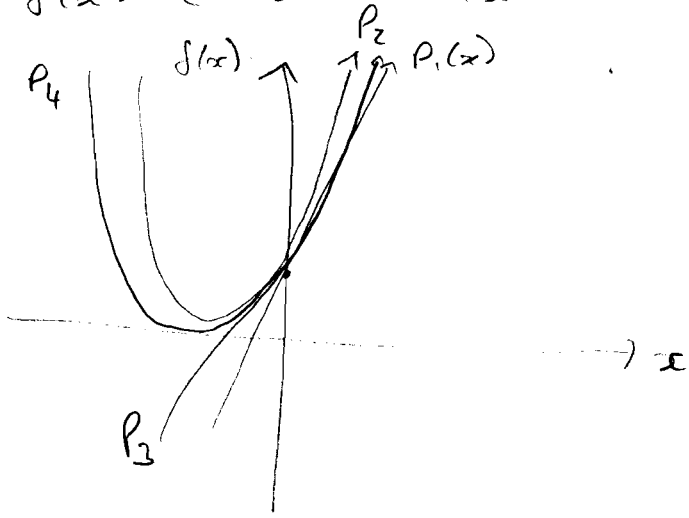
$P_2(.1) = .095$ $\quad P_2(.3) = .255$

$P_3(.1) = .095\overline{3}$ $\quad P_3(.3) = .255\overline{3}$

$P_4(.1) = .0953083\overline{3}$ $\quad P_4(.3) = .2553083\overline{3}$

a) $f(x) = (1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

b)



c) $P_1(.1) = 1.4$ $\quad P_1(.3) = 2.2$

$P_2(.1) = 1.46$ $\quad P_2(.3) = 2.74$

$P_3(.1) = 1.1464$ $\quad P_3(.3) = 2.848$

$P_4(.1) = 1.4641$ $\quad P_4(.3) = 2.8561$

(8) a) y-intersect of $2 + 3x - \frac{1}{2}x^2$ is 2 so impossible

b) $f'(x)$ is negative at $x = 0$ for $-1 - 5x + 2x^2$, from graph we see it is positive

c) from graph, the change of slope at $x = 0$ is positive; f'' of $-2 + 2x - \frac{1}{3}x^2$ is negative

(12)
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 + \cdots$$

Integrate $\ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$