2. $\int_1^3 (1 + 2x - 4x^3)\,dx = \left[x + 2\cdot\frac{1}{2}x^2 - 4\cdot\frac{1}{4}x^4\right]_1^3 = \left[x + x^2 - x^4\right]_1^3$

$$= (3 + 9 - 81) - (1 + 1 - 1) = -69 - 1 = -70$$

50. By the Net Change Theorem, $\int_0^{15} n'(t)\,dt = n(15) - n(0) = n(15) - 100$ represents the increase in the bee population in 15 weeks. So $100 + \int_0^{15} n'(t)\,dt = n(15)$ represents the total bee population after 15 weeks.

4. $A = \int_0^3 \left[(2y - y^2) - (y^2 - 4y)\right]\,dy = \int_0^3 (-2y^2 + 6y)\,dy$

$$= \left[-\tfrac{2}{3}y^3 + 3y^2\right]_0^3 = (-18 + 27) - 0 = 9$$

15. $1/x = x \iff 1 = x^2 \iff x = \pm 1$ and $1/x = \frac{1}{4}x \iff 4 = x^2 \iff x = \pm 2$, so for $x > 0$,

$$A = \int_0^1 \left(x - \frac{1}{4}x\right)dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right)dx = \int_0^1 \left(\frac{3}{4}x\right)dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right)dx = \left[\frac{3}{8}x^2\right]_0^1 + \left[\ln|x| - \frac{1}{8}x^2\right]_1^2$$

$$= \frac{3}{8} + \left(\ln 2 - \frac{1}{2}\right) - \left(0 - \frac{1}{8}\right) = \ln 2$$
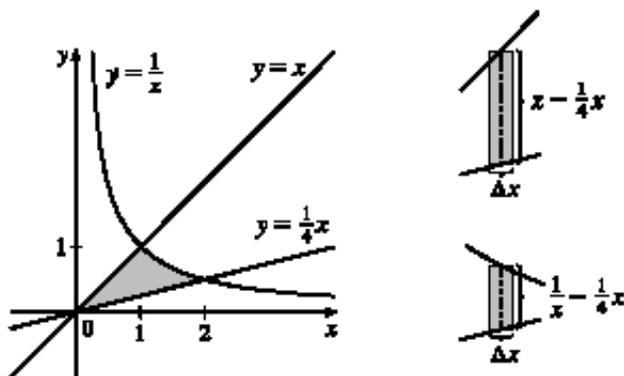
40. (a) We want to choose a so that

$$\int_1^a \frac{1}{x^2}\,dx = \int_a^4 \frac{1}{x^2}\,dx \quad\Rightarrow\quad \left[\frac{-1}{x}\right]_1^a = \left[\frac{-1}{x}\right]_a^4 \quad\Rightarrow\quad -\frac{1}{a}+1 = -\frac{1}{4}+\frac{1}{a} \quad\Rightarrow\quad \frac{5}{4} = \frac{2}{a} \quad\Rightarrow\quad a = \frac{8}{5}.$$

(b) The area under the curve $y = 1/x^2$ from $x = 1$ to $x = 4$ is $\frac{3}{4}$ [take $a = 4$ in the first integral in part (a)]. Now the line $y = b$ must intersect the curve $x = 1/\sqrt{y}$ and not the line $x = 4$, since the area under the line $y = 1/4^2$ from $x = 1$ to $x = 4$ is only $\frac{3}{16}$, which is less than half of $\frac{3}{4}$. We want to choose b so that the upper area in the diagram is half of the total area under the curve $y = 1/x^2$ from $x = 1$ to $x = 4$. This implies that

$\int_b^1 (1/\sqrt{y} - 1)\,dy = \frac{1}{2}\cdot\frac{3}{4} \quad\Rightarrow\quad \left[2\sqrt{y} - y\right]_b^1 = \frac{3}{8} \quad\Rightarrow\quad 1 - 2\sqrt{b} + b = \frac{3}{8} \quad\Rightarrow$

$b - 2\sqrt{b} + \frac{5}{8} = 0.$ Letting $c = \sqrt{b}$, we get $c^2 - 2c + \frac{5}{8} = 0 \quad\Rightarrow$

$8c^2 - 16c + 5 = 0.$ Thus, $c = \frac{16 \pm \sqrt{256 - 160}}{16} = 1 \pm \frac{\sqrt{6}}{4}.$ But $c = \sqrt{b} < 1 \quad\Rightarrow$

$c = 1 - \frac{\sqrt{6}}{4} \quad\Rightarrow\quad b = c^2 = 1 + \frac{3}{8} - \frac{\sqrt{6}}{2} = \frac{1}{8}\left(11 - 4\sqrt{6}\right) \approx 0.1503.$
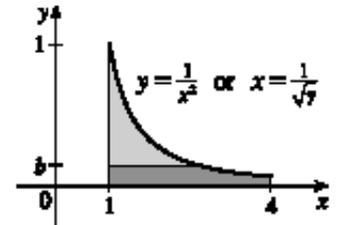
Integration Handout B

1

The two curves $y = -x^2 + 2$ and $y = x$ intersect at $x = -2$ and $x = 1$. Thus the area enclosed in these two curve is

$$\int_{-2}^{1} (-x^2 + 2 - x) \ dx.$$

3

(a) Is 0 because it is an odd function, therefore is symmetric around the origin.

(b)
$$\int_0^1 \arctan x \ dx = \int_0^{\frac{\pi}{4}} 1 - \tan y \ dy$$
$$= \int_0^{\frac{\pi}{4}} 1 \ dy - \int_0^{\frac{\pi}{4}} \tan y \ dy$$
$$= y]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{\sin y}{\cos y} \ dy \qquad \text{let } u = \cos y \text{ so } du = -\sin y \ dy$$
$$= \frac{\pi}{4} + \int_1^{\frac{\sqrt{2}}{2}} \frac{du}{u} = \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2}$$

6

(a) To approximate the amount of cobalt used, divide the block into n rectangles horizontally. For each rectangle, multiply the average density by the area. Then take the sum:

(b) $\int_0^6 \rho(x) \cdot 14 \ dx = 14 \int_0^6 \rho(x) \ dx.$

7

(a) To approximate the amount of cobalt used, divide the block into n rectangles horizontally. For each rectangle, multiply the average density by the area. Then take the sum and double it.

(b) $2 \int_0^3 \rho(x) \cdot 14 \ dx = 28 \int_0^3 \rho(x) \ dx.$

8

(a) To approximate the amount of cobalt used, slice the disc into n concentric rings. For each ring, multiply the average density by the circumference by the width. Then take the sum.

(b) $\int_0^8 \rho(x) \cdot 2\pi x \ dx = 2\pi \int_0^8 x\rho(x) \ dx.$

9 $r = 8 \Rightarrow x^2 + y^2 = 64 \Leftrightarrow y^2 = 64 - x^2 \Leftrightarrow y = \pm\sqrt{64 - x^2}$ So the amount of cobalt used is

$$2 \int_0^8 \rho(x) \cdot 2\sqrt{64 - x^2} \; dx = 4 \int_0^8 \sqrt{64 - x^2}\rho(x) \; dx.$$

10 The amount of holes is

$$\int_0^{10} \frac{1010}{\pi(x^2 + 1)} \cdot 2\pi x \; dx = 2020 \int_0^{10} \frac{x}{x^2 + 1} \; dx$$

$$= 1010[\ln(x^2 + 1)]_0^{10} = 1010 \cdot \ln(101)$$