

## SUMMARY OF SECTION 1.6

THEOREM 1 - EVERY LINEAR SYSTEM HAS EITHER NO SOLUTIONS, EXACTLY ONE SOLUTION OR INFINITELY MANY SOLUTIONS.

THEOREM 2 - IF  $A$  IS AN INVERTIBLE  $N \times N$  MATRIX AND  $b$  IS AN  $N \times 1$  MATRIX, THE SYSTEM  $Ax = b$  HAS EXACTLY ONE SOLUTION, NAMELY  $x = A^{-1}b$ .

THEOREM 3 -  $BA = I$ ,  $B$  SQUARE  $\Rightarrow B = A^{-1}$   
 $AB = I$ ,  $B$  SQUARE  $\Rightarrow B = A^{-1}$

THEOREM 4 - IF  $A$  IS  $N \times N$ , THE FOLLOWING STATEMENTS ARE EITHER ALL TRUE, OR ALL FALSE.

- a)  $A$  IS INVERTIBLE
- b)  $Ax = 0$  HAS ONLY THE TRIVIAL SOLUTION
- c) REDUCED ROW-ECHELON FORM OF  $A$  IS  $I_N$
- d)  $A = E_1 E_2 \cdots E_k$  FOR ELEMENTARY MATRICES  $E_1, E_2, \dots, E_k$ .
- e)  $Ax = b$  IS CONSISTENT FOR EVERY  $N \times 1$  MATRIX  $b$ .
- f)  $Ax = b$  HAS EXACTLY ONE SOLUTION FOR EVERY  $N \times 1$   $b$ .

THEOREM 5 - IF  $A$  AND  $B$  ARE SQUARE MATRICES OF THE SAME SIZE AND  $AB$  IS INVERTIBLE, THEN  $A$  AND  $B$  ARE INVERTIBLE.