SUMMARY OF SECTION 3.3

BY THE <u>ANGLE BETWEEN $\vec{u}$ AND $\vec{v}$</u>, WE MEAN THE ANGLE θ DETERMINED BY $\vec{u}$ AND $\vec{v}$ THAT SATISFIES $0 \le \theta \le \pi$.

THE <u>DOT PRODUCT</u> OR <u>EUCLIDEAN INNER PRODUCT</u> $\vec{u} \cdot \vec{v}$ IS DEFINED BY

$$\vec{u} \cdot \vec{v} = \begin{cases} \|\vec{u}\| \, \|\vec{v}\| \cos\theta & \text{IF } \vec{u} \neq \vec{0} \text{ AND } \vec{v} \neq \vec{0} \\ 0 & \text{IF } \vec{u} = \vec{0} \text{ OR } \vec{v} = \vec{0} \end{cases}$$

USING THE LAW OF COSINES ONE CAN SHOW THAT

$$(u_1, u_2, u_3) \cdot (v_1, v_2, v_3) = u_1 v_1 + u_2 v_2 + u_3 v_3 \quad \text{AND}$$

$$(u_1, u_2) \cdot (v_1, v_2) = u_1 v_1 + u_2 v_2$$

IF $\vec{u}$ AND $\vec{v}$ ARE NONZERO VECTORS $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$

<u>THEOREM 1</u> – a) $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$; THAT IS, $\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

b) IF $\vec{u}$ AND $\vec{v}$ ARE NONZERO AND θ IS THE ANGLE BETWEEN THEM, THEN

θ IS ACUTE $\iff \vec{u} \cdot \vec{v} > 0$

θ IS OBTUSE $\iff \vec{u} \cdot \vec{v} < 0$

$\theta = \frac{\pi}{2} \iff \vec{u} \cdot \vec{v} = 0$

<u>PERPENDICULAR VECTORS</u> ARE ALSO CALLED <u>ORTHOGONAL VECTORS</u>.

WE SAY THAT THE ZERO VECTOR IS PERPENDICULAR TO ALL OTHER VECTORS. IF $\vec{u}$ AND $\vec{v}$ ARE PERPENDICULAR WE WRITE $\vec{u} \perp \vec{v}$. NOTE: $\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$.