SUMMARY OF SECTION 4.2

A **FUNCTION** IS A RULE f THAT ASSOCIATES WITH EACH ELEMENT IN A SET A ONE AND ONLY ONE ELEMENT IN A SET B. IF f ASSOCIATES THE ELEMENT b WITH THE ELEMENT a, THEN WE WRITE $b = f(a)$ AND SAY THAT b IS THE **IMAGE** OF a UNDER f OR THAT $f(a)$ IS THE **VALUE** OF f AT a. THE SET A IS CALLED THE **DOMAIN** OF f AND THE SET B IS CALLED THE **CODOMAIN** OF f. THE SUBSET OF B CONSISTING OF ALL POSSIBLE VALUES FOR f AS a VARIES OVER A IS CALLED THE **RANGE** OF f. IF A AND B ARE SETS OF REAL NUMBERS WE CALL f A REAL VALUED FUNCTION OF A REAL VARIABLE. TWO FUNCTIONS f_1 AND f_2 ARE EQUAL, WRITTEN $f_1 = f_2$, IF THEY HAVE THE SAME DOMAIN AND $f_1(a) = f_2(a)$ FOR ALL a IN THE DOMAIN.

A FUNCTION $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ IS CALLED A **LINEAR TRANSFORMATION** (**LINEAR OPERATOR** IF $m = n$) IF FOR SOME MATRIX A

$$T(\vec{x}) = A\vec{x}$$ FOR ALL $\vec{x}$ IN $\mathbb{R}^n$. THE MATRIX A IS CALLED THE **STANDARD MATRIX** FOR T AND T IS CALLED **MULTIPLICATION BY** A. IF WE WISH TO EMPHASIZE THAT A IS THE STANDARD MATRIX FOR T WE WRITE $T_A(\vec{x})$ INSTEAD OF JUST $T(\vec{x})$. WE USE THE NOTATION $[T]$ TO REFER TO THE STANDARD MATRIX FOR T.

WHEN A IS A MATRIX OF ALL ZEROS WE CALL T THE **ZERO TRANSFORMATION**, WHILE IF A IS AN IDENTITY MATRIX WE CALL T THE **IDENTITY OPERATOR**.