(22) (a)

(2 pts)

$$\begin{aligned} 2x_1 + x_2 - 3x_3 + x_4 &= 0 \\ 5x_2 + 4x_3 + 3x_4 &= 0 \\ x_3 + 2x_4 &= 0 \\ 3x_4 &= 0 \end{aligned} \qquad \begin{bmatrix} 2 & 1 & -3 & 1 \\ 0 & 5 & 4 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

By careful observation, we see that we can easily solve the system by backwards substitution (starting with $3x_4 = 0$) to obtain the system's only solution, the trivial solution ($x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0$). By Theorem 1.6.4, we can conclude that the matrix (which is the system's coefficient matrix) is invertible.

(b)

$$\begin{aligned} 5x_1 + x_2 + 4x_3 + x_4 &= 0 \\ 2x_3 - x_4 &= 0 \\ x_3 + x_4 &= 0 \\ 7x_4 &= 0 \end{aligned} \qquad \begin{bmatrix} 5 & 1 & 4 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

It is clear by backward substitution that we obtain $x_3 = 0$, $x_4 = 0$. But the solutions to the system still must satisfy $5x_1 + x_2 = 0$ (we can't explicitly solve for a unique x_1, x_2). So there are non trivial solutions to the system. By Theorem 1.6.4, we can conclude the corresponding coefficient matrix is <u>not</u> invertible.