Math 20 Solution Set 11.8 HOMEWORK 24

1a. From equation 2, the expected payoff of the game is

$$pAq = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -4 & 6 & -4 & 1 \\ 5 & -7 & 3 & 8 \\ -8 & 0 & 6 & -2 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} = -5/8$$

b. If player R uses strategy $[p_1\ p_2\ p_3]$ against player C's strategy $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}^t$, his payoff will be $pAq = (\frac{-1}{4})p_1 + (9/4)p_2 - p_3$. Since p_1, p_2, p_3 are non-negative numbers and add up to 1, this is a weighted average of numbers $-1/4$, $9/4$, and -1. Clearly, this is largest if $p_1 = p_3 = 0$ and $p_2 = 1$. so $p = [0\ 1\ 0]$

c. As in (b), if player C uses $[q_1\ q_2\ q_3\ q_4]^t$ against $[1/2\ 0\ 1/2]$,

$pAq = -6q_1 + 3q_2 + q_3 - 1/2 q_4$. This is minimized by setting $q_1 = 1$ and $q_2 = q_3 = q_4 = 0$. So, $q = [1\ 0\ 0\ 0]^t$

3a. Calling the matrix A, we see a_{22} is a saddle point, so the optimal strategies are pure; $p = [0\ 1]$, $q = [0\ 1]^t$; the value of the game is $a_{22} = 3$.

b. As in (a), a_{21} is a saddle point, so optimal strategies are $p = [0\ 1\ 0]$, $q = [1\ 0]^t$; the value of the game is $a_{21} = 2$.

c. Here, a_{32} is a saddle point, so optimal strategies are $p = [0\ 0\ 1]$, $q = [0\ 1\ 0]^t$ and $v = a_{32} = 2$.

d. Here, a_{21} is a saddle point, so $p = [0\ 1\ 0\ 0]$, $q = [1\ 0\ 0]^t$ and $v = a_{21} = -2$.

4a. Calling the matrix A, the formula from Theorem 2 on P.605 yields

$p = [5/8\ \ 3/8]$, $q = [1/8\ \ 7/8]^t$, $v = 27/8$. A has no saddle points.