4b. As in (a), use Theorem 11.8.2. $p = \left[\frac{40}{60} \;\; \frac{20}{60}\right] = \left[\frac{2}{3} \;\; \frac{1}{3}\right]$,

$q = \left[\frac{10}{60} \;\; \frac{50}{60}\right]^t = \left[\frac{1}{6} \;\; \frac{5}{6}\right]^t$, $v = \frac{1400}{60} = \frac{70}{3}$ (A has no saddle points).

c. a_{11} is a saddle point, so $p = [1 \;\; 0]$, $q = [1 \;\; 0]^t$ and $v = a_{11} = 3$.

d. This matrix has no saddle points, so again use Thm 11.8.2.

$p = \left[\frac{3}{5} \;\; \frac{2}{5}\right]$, $q = \left[\frac{3}{5} \;\; \frac{2}{5}\right]^t$, $v = \frac{19}{5}$

e. A has no saddle point, so use Thm 11.8.2. $p = \left[\frac{3}{13} \;\; \frac{10}{13}\right]$

$q = \left[\frac{1}{13} \;\; \frac{12}{13}\right]$, $v = \frac{-29}{13}$.

5. Let a_{11} = payoff to R if the black ace and black two are played = 3
a_{12} = payoff to R if the black ace and red three are played = −4
a_{21} = payoff to R if the red four and the black two are played = −6
a_{22} = payoff to R if the red four and red three are played = 7

So, the payoff matrix for the game is $\begin{bmatrix} 3 & -4 \\ -6 & 7 \end{bmatrix} = A$

A has no saddle points, so from Theorem 2, $p = \left[\frac{13}{20} \;\; \frac{7}{20}\right]$,

$q = \left[\frac{11}{20} \;\; \frac{9}{20}\right]^t$. So, player R should play the Black ace 65% of the time, and player C should play the black two 55% of the time. The value of the game is $-3/20$, so player C can expect to collect on the average of fifteen cents per game.