

Math 20 - Spring 2002
 Homework # 28 Solutions

Handout Sec. 6 / # 4, 8, 11, 16, 19, 26, 28, 34

4. $f(x,y) = \sin(x+2y)$. at $(4,-2)$ in direction $\theta = \frac{3\pi}{4}$.

$$\Rightarrow \nabla f(x,y) = \left(\frac{\partial f}{\partial x} \Big|_{(x,y)}, \frac{\partial f}{\partial y} \Big|_{(x,y)} \right) = (\cos(x+2y), 2\cos(x+2y))$$

$$\nabla f(4,-2) = (\cos 0, 2\cos 0) = (1, 2)$$

Let \vec{u} be a unit vector in direction $\theta = \frac{3\pi}{4}$ from the x-axis $\Rightarrow \vec{u} = \left(\cos \frac{3\pi}{4}, \sin \frac{3\pi}{4} \right)$

$$\Rightarrow D_{\vec{u}} f(4,-2) = \nabla f(4,-2) \cdot \vec{u} = (1, 2) \cdot \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2}$$

$$= \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$$

$$D_{\vec{u}} f(4,-2) = \frac{\sqrt{2}}{2}$$

8. $f(x,y) = y \ln x$ $P(1,-3)$ $u = \left(-\frac{4}{5}, \frac{3}{5} \right)$

(a) $\nabla f(x,y) = \left(\frac{y}{x}, \ln x \right)$

(b) $\nabla f(1,-3) = (-3, 0)$

(c) $D_{\vec{u}} f(1,-3) = \nabla f(1,-3) \cdot \vec{u} = \frac{12}{5}$

\vec{u} is a unit vector:
 $\|\vec{u}\| = \sqrt{\left(-\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = 1$

11. $f(x,y) = 1 + 2x\sqrt{y}$ $(3,4)$ $\vec{v} = \langle 4, -3 \rangle$

$$\nabla f(x,y) = (2\sqrt{y}, xy^{-\frac{1}{2}}) \Rightarrow \nabla f(3,4) = \left(4, \frac{3}{2} \right)$$

$$D_{\vec{v}} f(3,4) = \nabla f(3,4) \cdot \frac{\vec{v}}{\|\vec{v}\|} = \left(4, \frac{3}{2} \right) \cdot \left(\frac{4}{5}, -\frac{3}{5} \right) = \frac{16}{5} - \frac{9}{10}$$

\uparrow
 since \vec{v} is not
 a unit vector

$$\Rightarrow D_{\vec{v}} f(3,4) = \frac{23}{10}$$