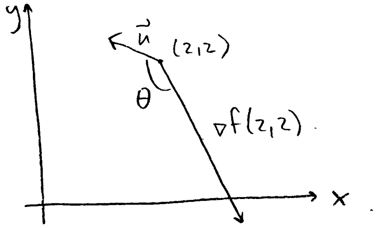
16.



$\theta \approx 150^\circ$

$\|\nabla f(2,2)\| \approx 3$ (using length of $\vec{u}$ as yardstick)

$$D_{\vec{u}} f(2,2) = \nabla f(2,2) \cdot \vec{u}$$
$$= \|\nabla f(2,2)\| \, \overset{1}{\cancel{\|\vec{u}\|}} \cos\theta$$
$$\approx 3 \cdot \cos 150^\circ$$
$$\approx -\frac{3\sqrt{3}}{2}$$

19. $f(x,y) = \sin(xy)$ $\quad P(1,0)$

The maximum rate of change at (x_0, y_0) is $\|\nabla f(x_0, y_0)\|$.

$\Rightarrow$ maximum rate of change $= \|\nabla f(1,0)\|$

$$= \left\| \left(y\cos(xy),\ x\cos(xy) \right)\Big|_{(1,0)} \right\|$$

$$= \|(0,1)\| \Rightarrow \boxed{\text{the direction of max increase} = (0,1)\ (= \nabla f(1,0))}$$

$$\boxed{\text{max. rate of change} = 1}$$

26 $\quad z = 200 + 0.02x^2 - 0.001y^3$

$\nabla z(x,y) = (0.04x, -0.003y^2)$

$\nabla z(80,60) = (3.2, -10.8)$

The boat is traveling in the direction $\vec{v} = (-80,-60) \Rightarrow \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \left(-\frac{4}{5}, -\frac{3}{5}\right)$

$$\Rightarrow D_{\vec{u}} z(80,60) = \nabla z(80,60) \cdot \vec{u}$$
$$= (3.2, -10.8) \cdot \left(-\frac{4}{5}, -\frac{3}{5}\right)$$
$$= 3.92$$

So the water is getting <u>deeper</u> as the boat departs.