7. Let $x = (x_1, x_2, x_3)$

one side of eqn: $2u - v + x = (-6,2,4) - (4,0,-8) + (x_1, x_2, x_3) = (-10 + x_1, 2 + x_2, 12 + x_3)$

other side: $7x + w = 7(x_1, x_2, x_3) + (6,-1,-4) = (7x_1 + 6, 7x_2 - 1, 7x_3 - 4)$

Solve:
$$-10 + x_1 = 7x_1 + 6$$
$$2 + x_2 = 7x_2 - 1$$
$$12 + x_3 = 7x_3 - 4$$

$x = (-8/3, 1/2, 8/3)$

8. Find $c_1, c_2, c_3$ st $c_1 u + c_2 v + c_3 w = (2,0,4)$

$$c_1 u + c_2 v + c_3 w = (-3c_1, c_1, 2c_1) + (4c_2, 0, -8c_2) + (6c_3, -1c_3, -4c_3)$$
$$= (-3c_1 + 4c_2 + 6c_3, c_1 - c_3, 2c_1 - 8c_2 - 4c_3)$$

3 eqns!
$$-3c_1 + 4c_2 + 6c_3 = 2$$
$$c_1 - c_3 = 0 \Rightarrow c_1 = c_3$$
$$2c_1 - 8c_2 - 4c_3 = 4$$

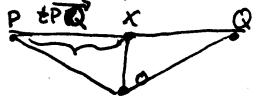
So,
$$4c_2 + 3c_3 = 2$$
$$-8c_2 - 2c_3 = 4 \Rightarrow \begin{bmatrix} 4 & 3 \\ -8 & -2 \end{bmatrix}\begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ -8 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} -1/8 & -3/16 \\ 1/2 & 1/4 \end{bmatrix}, \text{ so } \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -1/8 & -3/16 \\ 1/2 & 1/4 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

So $(c_1, c_2, c_3) = (2, -1, 2)$.

10. If we equate both sides of the given equation, we get:
$$c_1 + 2c_2 = 0$$
$$2c_1 + c_2 + 3c_3 = 0$$
$$c_2 + c_3 = 0.$$
The augmented matrix of these equations can be reduced to $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,
so the only solution is $c_1 = c_2 = c_3 = 0$.

11. We work in the plane determined by the origin, P and Q. Let X be a point on the line through P and Q and let $t\overrightarrow{PQ}$ (where t is a positive real #) be the vector with initial point P and terminal point X. From the figure below, we see that
$\overrightarrow{OP} + t\,\overrightarrow{PQ} = \overrightarrow{OX}$ and $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$.




Therefore $\overrightarrow{OX} = \overrightarrow{OP} + t(\overrightarrow{OQ} - \overrightarrow{OP}) = (1-t)\overrightarrow{OP} + t\,\overrightarrow{OQ}$

a. To obtain the midpoint of the line segment connecting P and Q, we set $t = 1/2$.
This gives, $\overrightarrow{OX} = \frac{1}{2}\overrightarrow{OP} + \frac{1}{2}\overrightarrow{OQ} = \frac{1}{2}(2,3,-2) + \frac{1}{2}(7,-4,1) = (\frac{9}{2}, \frac{-1}{2}, \frac{-1}{2})$

b. setting $t = 3/4$ gives $\overrightarrow{OX} = \frac{1}{4}(2,3,-2) + \frac{3}{4}(7,-4,1) = (\frac{23}{4}, \frac{-9}{4}, \frac{1}{4})$