11a. (cont'd)

Now we need a point on this line of intersection (a point that satisfies both S_1 and S_2). Set $z=0$ and solve the resulting equations:

$$\left.\begin{aligned} 7x-2y &= -2 \\ -3x+y &= -5 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= -12 \\ y &= -41. \end{aligned}$$

So $(-12,-41,0)$ is a point on the line.

$\Rightarrow$ $\boxed{\text{line of intersection: } x = -12-7t,\ y = -41-23t,\ z = t}$

14a. $S_1: (-2,1,4)\cdot(x-1,y,z+3) = 0 \Rightarrow \vec{n}_1 = (-2,1,4)$

(0.5 pts) $S_2: (1,-2,1)\cdot(x+3,y-5,z) = 0. \Rightarrow \vec{n}_2 = (1,-2,1)$

$\Rightarrow \vec{n}_1\cdot\vec{n}_2 = 0$

Since the normal vectors of S_1 and S_2 are perpendicular, $\boxed{S_1 \perp S_2}$

17. (1pt)

line: $\begin{cases} x = 4+2t \\ y = -2+3t \\ z = -5t \end{cases}$

point $(-2,1,7)$.

The direction vector of the line $(2,3,-5)$ gives us the normal vector of the plane we are looking for. So,

$$(2,3,-5)\cdot(x+2, y-1, z-7) = 0$$
$$2(x+2) + 3(y-1) - 5(z-7) = 0$$
$$\Rightarrow \boxed{2x+3y-5z+36=0}$$

25. (1pt) $P_1(-1,-2,-3)$ $P_2(-2,0,1)$ $P_3(-4,-1,-1)$ $P_4(2,0,1)$.

Assume P_1, P_2, P_3 are in the same plane (three points uniquely from a plane). Then we need to show P_4 lies in that plane.

$$\left.\begin{aligned} \overrightarrow{P_1P_2} &= (-1,2,4) \\ \overrightarrow{P_1P_3} &= (-3,1,2) \end{aligned}\right\} \Rightarrow \vec{n} = \overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3} = (0,10,-5)$$

So the plane formed by P_1, P_2, P_3 is $10(y+2)-5(z+3)=0$.

Does P_4 lie in this plane? Substitute P_4 in the formula for the plane:

$$10(0+2) - 5(1+3) = 10(2) - 5(4) = 20-20 = 0.$$

So P_4 lies in the plane also $\Rightarrow$ $\boxed{P_1, P_2, P_3, P_4 \text{ are coplanar}}$