HW 15 4.1 # 3, 4, 5d, 9d, 11d, 15b, 16, 20, 21, 23

TOTAL = 8 pts

③ 1pt

$$\begin{bmatrix} -1 & 2 & 7 & 6 \\ 3 & 0 & 1 & 3 \\ 2 & 4 & 1 & 1 \\ 0 & -1 & 4 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 6 \\ -3 \end{bmatrix}$$

augmented matrix

$$\left[\begin{array}{cccc|c} -1 & 2 & 7 & 6 & 0 \\ 3 & 0 & 1 & 3 & 5 \\ 2 & 4 & 1 & 1 & 6 \\ 0 & -1 & 4 & 2 & -3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} -1 & 2 & 7 & 6 & 0 \\ 0 & 6 & 22 & 21 & 5 \\ 0 & 8 & 15 & 13 & 6 \\ 0 & -1 & 4 & 2 & -3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} -1 & 0 & 15 & 10 & -6 \\ 0 & 0 & 46 & 33 & -13 \\ 0 & 0 & 47 & 29 & -18 \\ 0 & -1 & 4 & 2 & -3 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -15 & -10 & 6 \\ 0 & 1 & -4 & -2 & 3 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 46 & 33 & -13 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -70 & -69 \\ 0 & 1 & 0 & -18 & -17 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 217 & 217 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

thus, $\boxed{c_1 = 1,\ c_2 = 1,\ c_3 = -1,\ c_4 = 1}$

④ 1pt

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 3 \end{bmatrix}$$

Notice that when written in this form we can clearly see that the 2nd row is a row of zeros. If we were to examine this further we would get: $0c_1 + 0c_2 + 0c_3 = -2$ which is a contradiction, $0 \neq -2$. Thus, scalars $c_1, c_2,$ and c_3 do not exist to satisfy the original equation.

⑤ d. .5 pt

$$\|\vec{u}\| = \sqrt{(-2)^2 + (1)^2 + (1)^2 + (-3)^2 + (4)^2} = \sqrt{4+1+1+9+16} = \boxed{\sqrt{31}}$$

⑨ d. .5 pt

$$\vec{u} \cdot \vec{v} = (-1, 1, 0, 4, -3) \cdot (-2, -2, 0, 2, -1) = (-2)(-1) + (1)(-2) + (0)(0) + (4)(2) + (-3)(-1)$$
$$= 2 + {}^-2 + 0 + 8 + 3 = \boxed{11}$$

⑪ d. .5 pt

$$\|\vec{u} - \vec{v}\| = \sqrt{(3-(-4))^2 + (-3-1)^2 + (-2-(-1))^2 + (0-5)^2 + (-3-0)^2} = \sqrt{49+16+1+25+9} = \sqrt{100} = \boxed{10}$$

⑮ b. .5 pt

orthogonal $\Leftrightarrow \vec{u} \cdot \vec{v} = 0$

$$\vec{u} \cdot \vec{v} = (k, k, 1) \cdot (k, 5, 6) = k^2 + 5k + 6 = (k+2)(k+3) = 0 \Rightarrow \boxed{k = -2, -3}$$