(16) orthogonal $\Leftrightarrow \vec{a}\cdot\vec{b}=0$

$\boxed{1pt}$ let $\vec{x} = (x_1, x_2, x_3, x_4)$

$\vec{x}\cdot\vec{u} = (x_1,x_2,x_3,x_4)\cdot(2,1,-4,0) = 2x_1+x_2-4x_3=0$

$\vec{x}\cdot\vec{v} = (x_1,x_2,x_3,x_4)\cdot(-1,-1,2,2) = -x_1-x_2+2x_3+2x_4=0$

$\vec{x}\cdot\vec{w} = (x_1,x_2,x_3,x_4)\cdot(3,2,5,4) = 3x_1+2x_2+5x_3+4x_4=0$

augmented matrix:

$$\left[\begin{array}{cccc|c} 2 & 1 & -4 & 0 & 0 \\ -1 & -1 & 2 & 2 & 0 \\ 3 & 2 & 5 & 4 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ -1 & -1 & 2 & 2 & 0 \\ 0 & -1 & 11 & 10 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & -1 & 0 & 4 & 0 \\ 0 & -1 & 11 & 10 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 11 & 6 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 1 & 0 & -\frac{17}{3} & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 11 & 6 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 - \frac{17}{3}x_3 = 0 \Rightarrow \\ x_2 - 4x_4 = 0 \Rightarrow \\ 11x_3 + 6x_4 = 0 \Rightarrow \end{array} \left.\begin{array}{l} x_3 = \frac{3}{17}x_1 \\ x_2 = 4x_4 = -\frac{22}{17}x_1 \\ x_4 = -\frac{11}{6}x_3 = -\frac{11}{34}x_1 \end{array}\right\} \left(1, -\frac{22}{17}, \frac{3}{17}, -\frac{11}{34}\right)$$

$\vec{x} = (34, -44, 6, -11)$, $\|\vec{x}\| = \sqrt{34^2+(-44)^2+6^2+(-11)^2} = \sqrt{3249} = 57$

answer: $\boxed{\pm\frac{1}{57}(34,-44,6,-11)}$

(20) $\|\vec{u}+\vec{v}\| = 1$ and $\|\vec{u}-\vec{v}\| = 5$

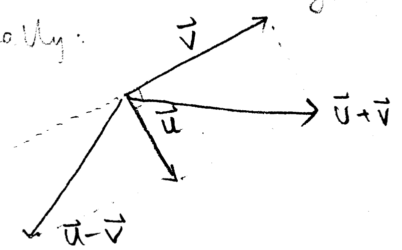
$\boxed{1pt}$ Theorem 4.1.6: $\vec{u}\cdot\vec{v} = \frac{1}{4}\|\vec{u}+\vec{v}\|^2 - \frac{1}{4}\|\vec{u}-\vec{v}\|^2 = \frac{1}{4}(1)^2 - \frac{1}{4}(5)^2 = \frac{1}{4} - \frac{25}{4} = \boxed{-6}$

(21) $\vec{u}\cdot\vec{v} = \frac{1}{4}\|\vec{u}+\vec{v}\|^2 - \frac{1}{4}\|\vec{u}-\vec{v}\|^2$

$\boxed{1pt}$ given $\|\vec{u}+\vec{v}\| = \|\vec{u}-\vec{v}\|$ we get $\vec{u}\cdot\vec{v} = \frac{1}{4}\|\vec{u}+\vec{v}\| - \frac{1}{4}\|\vec{u}+\vec{v}\| = 0$

thus $\vec{u}$ and $\vec{v}$ are orthogonal

graphically:



if $\|\vec{u}+\vec{v}\| = \|\vec{u}-\vec{v}\|$ then graphically we can see that $\vec{u}$ must be orthogonal to $\vec{v}$

(23) set the two parameterizations equal

$\boxed{1pt}$ $\vec{r}_1 = (3,2,3,-1) + t_1(4,6,4,-2)$ $\qquad$ $\vec{r}_2 = (0,3,5,4) + t_2(1,-3,-4,-2)$

$3+4t_1 = 0+1t_2$

$2+6t_1 = 3-3t_2$

$3+4t_1 = 5-4t_2$

$-1-2t_1 = 4-2t_2$

examine these two equations...

$3+4t_1 = t_2 = 5-4t_2 \Rightarrow 5t_2 = 5 \Rightarrow \underline{t_2 = 1}$

plug back in: $3+4t_1 = 1 \Rightarrow \underline{t_1 = -\frac{1}{2}}$

is this consistent?

does: $2+6(-\frac{1}{2}) = 3-3(1)$? $\quad -1=0$ ✗

does: $-1-2(-\frac{1}{2}) = 4-2(1)$? $\quad 0=2$ ✗

$\boxed{\text{thus, they do not intersect in } R^4}$