

$$13a. T(-2, 1, 2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{\sqrt{3}}{2} - 1 \\ \frac{1}{2} + \sqrt{3} \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{\sqrt{3}-2}{2} \\ \frac{1+2\sqrt{3}}{2} \end{bmatrix}$$

$$b. T(-2, 1, 2) = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2\sqrt{2} \end{bmatrix}$$

$$c. T(-2, 1, 2) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$$

$$17a. \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$b. \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$$

$$c. \begin{bmatrix} \cos(15+105+60) & -\sin(15+105+60) \\ \sin(15+105+60) & \cos(15+105+60) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

21. a. Geometrically, it doesn't make any difference whether we rotate and then dilate or whether we dilate and then rotate. A dilation/contraction is represented as a scalar multiple of the ID matrix. Since such a matrix commutes with any square matrix of appropriate size, the transformations commute.

$$b. T_1 \circ T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \cos \theta_1 \cos \theta_2 & \cos \theta_1 \sin \theta_2 & -\sin \theta_1 \\ \sin \theta_2 \sin \theta_1 & \sin \theta_1 \cos \theta_2 & \cos \theta_1 \end{bmatrix}$$

$$T_2 \circ T_1 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix} = \begin{bmatrix} \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 \sin \theta_2 \\ \sin \theta_2 & \cos \theta_1 \cos \theta_2 & -\sin \theta_1 \cos \theta_2 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix}$$

Clearly, the transformations do not commute.