Solutions for Homework 20

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9.3 / #1, 4, 8

6 pts total

① least squares straight line fit for: $(0,0), (1,2), (2,7)$

2pts

$$M = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \quad M^T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad M^TM = \begin{bmatrix} 3 & 3 \\ 3 & 5 \end{bmatrix} \quad (M^TM)^{-1} = \frac{1}{15-9}\begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix} \quad y = \begin{bmatrix} 0 \\ 2 \\ 7 \end{bmatrix}$$

$$v^* = (M^TM)^{-1}M^Ty = \frac{1}{6}\begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{7}{2} \end{bmatrix}$$

least squares straight line: $\boxed{y = -\frac{1}{2} + \frac{7}{2}x}$

④ cubic polynomial that best fits: $(-1,-14), (0,-5), (1,-4), (2,1), (3,22)$

2pts

$$M = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix} \quad M^TM = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 4 & 9 \\ -1 & 0 & 1 & 8 & 27 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 15 & 35 \\ 5 & 15 & 35 & 99 \\ 15 & 35 & 99 & 275 \\ 35 & 99 & 275 & 795 \end{bmatrix}$$

$$(M^TM)^{-1} = \begin{bmatrix} \frac{27}{35} & -\frac{1}{42} & -\frac{4}{7} & \frac{1}{6} \\ -\frac{1}{42} & \frac{25}{63} & -\frac{5}{84} & -\frac{1}{36} \\ -\frac{4}{7} & -\frac{5}{84} & \frac{39}{56} & -\frac{5}{24} \\ \frac{1}{6} & -\frac{1}{36} & -\frac{5}{24} & \frac{5}{12} \end{bmatrix} \quad \text{calculator} \qquad y = \begin{bmatrix} -14 \\ -5 \\ -4 \\ 1 \\ 22 \end{bmatrix}$$

$$v^* = (M^TM)^{-1}M^Ty = \begin{bmatrix} -5 \\ 3 \\ -4 \\ 2 \end{bmatrix} \quad \text{by calculator} \qquad \boxed{y = -5 + 3x - 4x^2 + 2x^3}$$

⑧ quadratic polynomial that best fits: $(1,4.0), (2,4.4), (3,5.2), (4,6.4), (5,8.0)$

2pts

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix} \quad M^TM = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 9 & 16 & 25 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix} = \begin{bmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix}$$

$$(M^TM)^{-1} = \begin{bmatrix} \frac{23}{5} & -\frac{33}{10} & \frac{1}{2} \\ -\frac{33}{10} & \frac{187}{70} & -\frac{3}{7} \\ \frac{1}{2} & -\frac{3}{7} & \frac{1}{14} \end{bmatrix} \quad \text{calculator} \qquad y = \begin{bmatrix} 4.0 \\ 4.4 \\ 5.2 \\ 6.4 \\ 8.0 \end{bmatrix}$$

$$v^* = (M^TM)^{-1}M^Ty = \begin{bmatrix} 4.0 \\ -.2 \\ .2 \end{bmatrix} \qquad \boxed{y = 4 - .2x + .2x^2}$$

$$y(12) = 4 - .2(12) + .2(12)^2 = \boxed{30.4}$$