16. a) Linear transformation from $R^2 \to R^3$, one-to-one

b) Linear transformation from $R^3 \to R^2$, not one-to-one.

18. a) this maps the x-axis to vectors themselves, the y-axis vectors to their negatives, and no other vectors are mapped to scalar multiples of themselves.
So $\lambda = \pm 1$, eigenvectors are $(x,0)$ or $(0,y)$.

The matrix representation of the transformation is

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad |\lambda I - [T]| = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda+1 \end{vmatrix} = (\lambda-1)(\lambda+1) = 0$$

so $\lambda = \pm 1$.

Let (x,y) be an eigenvector with eigen value $\lambda = 1$.
Then

$$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So $y=0$ or the vector must lie on the x-axis.
Similarly, $\lambda = -1$ implies

$$\begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2x \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So $x=0$ or vector lies on y-axis.

d) All vectors point in the same direction but have $1/2$ the length after the transformation.
so $\lambda = 1/2$.

$$[T] = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \qquad |\lambda I - [T]| = \begin{vmatrix} \lambda - 1/2 & 0 \\ 0 & \lambda - 1/2 \end{vmatrix} = (\lambda - 1/2)^2 = 0$$

so $\lambda = 1/2$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ for all } x \text{ and } y,$$

so all vectors are eigenvectors of $\lambda = 1/2$.