

Math 20 Fall 2003
Final Exam Practice Problems (Set 2)

1. Congressional elections are held every two years.
 - (a) Of the persons in a certain precinct voting Democratic in one election, 70% will vote Democratic, 20% will vote Republican, and 10% will vote Libertarian in the next election.
 - (b) Of the Republican voters in the precinct, 10% will vote Democratic, 80% will vote Republican, and 10% will vote Libertarian in the next election.
 - (c) Of the Libertarian voters in the precinct, 30% will vote Democratic, 30% will vote Republican, and 40% will vote Libertarian in the next election.

Assuming these voting patterns persist, what percentage of voters will vote Democratic, Republican, and Libertarian in an election many years from now?

2. Consider an economy with three sectors, Chemicals, Fuels, and Machinery.
 - (a) Chemicals sells 30% of its output to Fuels and 50% to Machinery and retains the rest.
 - (b) Fuels sells 80% of its output to Chemicals and 10% to Machinery and retains the rest.
 - (c) Machinery sells 40% of its output to Chemicals and 40% to Fuels and retains the rest.

Find a set of equilibrium prices when the price for the Machinery output is 100 million dollars.

3. A cell phone manufacturer is introducing two new lines of phones, the Standard and the Executive. Each phone needs a battery, a chip, and a plastic case.
 - (a) The Standard model uses Everlast I batteries, available in unlimited quantities from an outside supplier. The Executive model uses Everlast II batteries, 6,000 of which can be produced in-house each year.
 - (b) The Executive model uses Intel 60 chips, available in unlimited quantities from an outside supplier. The Standard model uses Intel 48 chips, 4,000 of which can be produced in-house each year.
 - (c) The Standard model's plastic case requires 3 ounces of plastic, while the Executive model's plastic case requires 2 ounces of plastic. The cell phone manufacturer has access to 18,000 ounces of plastic each year.
 - (d) Market research indicates that the Standard model can be sold for \$3 net profit, while the Executive model can be sold for \$5 net profit.

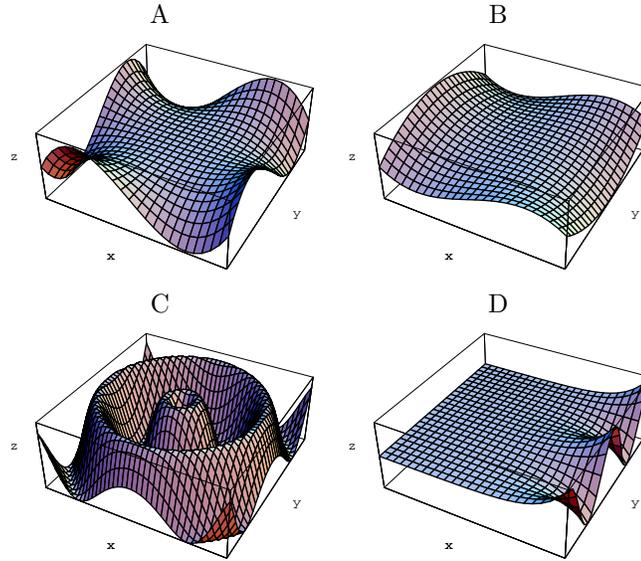
What product mix will maximize expected profit for the coming year?

4. Find an equation of the tangent plane to the surface $z = x^2 + xy + 3y^2$ at the point $(1, 1, 5)$.
5. Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin xy$ at the point $(1, 0)$ has the value 1.
6. Find the extreme values of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the region $\{(x, y) \mid x^2 + y^2 \leq 16\}$.

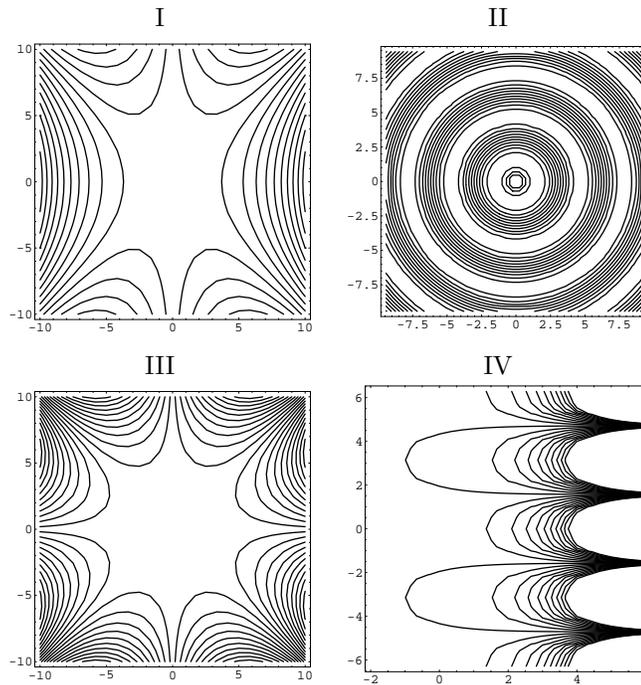
7. Match each of the following functions with its graph and with its contour map.

- (a) $f(x, y) = e^x \cos y$
- (b) $f(x, y) = xy^2 - x^3$
- (c) $f(x, y) = xy^3 - x^3y$
- (d) $f(x, y) = \sin \sqrt{x^2 + y^2}$

Graphs



Contour Maps



Answers

1. Democratic: $\frac{9}{28} \approx 32\%$; Republican: $\frac{15}{28} \approx 54\%$; Libertarian: $\frac{1}{7} \approx 14\%$
2. Chemicals: $\frac{425}{3} \approx 142.7$; Fuels: $\frac{275}{3} \approx 91.7$; Machinery: 100 (in millions of dollars)
3. 2,000 Standard models and 6,000 Executive models for \$36,000 profit
4. $z - 5 = 3(x - 1) + 7(y - 1)$
5. unit direction vectors $(0, 1)$ and $(\frac{4}{5}, -\frac{3}{5})$
6. absolute maximum of 47 at $(-2, \pm\sqrt{12})$, absolute minimum of -7 at $(1, 0)$
7. (a) D, IV
(b) B, I
(c) A, III
(d) C, II