§1.2 Gaussian Elimination
September 17, 2003

1 Homework

§1.2 #4, 5, 6, 7, 12, 13, 25, 27

2 Echelon Forms

A matrix that is in reduced row-echelon form satisfies all of the following properties.

1. If a row does not consist entirely of zeros, then the first nonzero number in the row is a 1. We call this a leading 1.

2. If there are any rows that consist entirely of zeros, then they are grouped together at the bottom of the matrix.

3. In any two successive rows that do not consist entirely of zeros, the leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row.

4. Each column that contains a leading 1 has zeros everywhere else.

A matrix that has the first three properties is said to be in row-echelon form.

3 Elimination Methods

The following elimination algorithm can be used to reduce any matrix to reduced row-echelon form.

1. Locate the leftmost column that does not consist entirely of zeros.

2. Interchange the top row with another row, if necessary, to bring a nonzero entry to the top of the column found in Step 1.

3. If the entry that is now at the top of the column found in Step 1 is a, multiply the first row by $\frac{1}{a}$ in order to introduce a leading 1.

4. Add suitable multiples of the top row to the rows below so that all entries below the leading 1 become zeros.

5. Now cover the top row in the matrix and begin again with Step 1 applied to the submatrix that remains. Continue this way until the entire matrix is in row-echelon form.

6. Beginning with the last nonzero row and working upward, add suitable multiples of each row to the rows above to introduce zeros above the leading 1's.

If we use only the first five steps of this algorithm, we get a matrix in row-echelon form and the procedure is called Gaussian elimination. If we use all six steps, we get a matrix in reduced row-echelon form and the procedure is called Gauss-Jordan elimination.

4 Using the Elimination Methods

Problem 1.2.1: Solve the following system of linear equations.

$$\begin{array}{ccccccc} x & + & 3y & + & z & = & 3 \\ 2x & + & 2y & + & 4z & = & -4 \\ 3x & + & y & - & z & = & 9 \end{array}$$

Problem 1.2.2: Solve the following system of linear equations.

$$\begin{array}{ccccc} 2x_1 & - & x_2 & + & 8x_3 = 3 \\ x_1 & + & 3x_2 & - & 3x_3 = 5 \\ -x_1 & + & 5x_2 & - & 13x_3 = 3 \end{array}$$