Problem 11.9.1 (Part 1): The economy of a small developing nation consists of three groups of producers. Farmers produce food, builders produce shelter, and tailors produce clothing.

Of the food produced by the farmers, $\frac{7}{16}$ is bought by the farmers themselves, $\frac{5}{16}$ by the builders, and $\frac{1}{4}$ by the tailors. Of the shelter produced by the builders, $\frac{1}{2}$ is bought by the farmers, $\frac{1}{6}$ by the builders themselves, and $\frac{1}{3}$ by the tailors. Of the clothing produced by the tailors, $\frac{3}{16}$ is bought by the farmers, $\frac{5}{16}$ by the builders, and $\frac{1}{2}$ by the tailors themselves.

Represent the above information in a matrix E, where the entry e_{ij} is the fraction of the total output of the jth industry purchased by the ith industry. (Label the farmers, builders, and tailors as industries 1, 2, and 3, respectively.)

(Part 2:) Let p_1 be the total value (in units of local currency) of all food produced by the farmers in a given year. (Thus p_1 is the total income received by the farmers from food sales, and, for example $\frac{5}{16}p_1$ is the amount the builders spend on food.) Likewise, let p_2 be the total value of all shelter produced by the builders and p_3 be the total value of all clothing produced by the tailors.

Set up (but do not solve) a homogeneous system of linear equations that answers the following question: What should p_1, p_2, and p_3 be so that this simple economy is in equilibrium, that is, so that the income of each group of producers is equal to the value of their expenses?

(Part 3:) Represent the homogeneous system of linear equations you found in Part 2 as a single matrix equation using the matrix E you found in Part 1 and the matrix

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}.$$