

§11.6 Markov Chains

Problem: Smallville is the only town in its mostly rural county in Kansas. It is found that every year about 20% of those living in the town of Smallville move to the surrounding countryside and about 30% of those living in the countryside move into the town of Smallville.

(a) Represent this Markov chain by a transition matrix.

(b) If 40% of the population of the county currently resides in the town of Smallville, what percentage of the population of the county is likely to live in Smallville 1 year from now? 2 years from now?

(c) What percentage of the county's population can be expected to sides in the town of Smallville after many years?

Solution:

(a)

$$P = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$$

(b) 50% in the town 1 year from now; 55% in the town 2 years from now

(c) 60% in the town many years from now

§11.8 Games of Strategy

Problem: Suppose that you have some amount of money to invest in stocks, bonds, and savings accounts. The following payoff matrix gives your expected return (as a percentage of money invested) in each of these categories in relation to the growth of the economy.

Action \ Growth	High	Medium	None	Negative
Bonds	12	8	6	3
Stocks	15	7	3	-2
Savings	7	7	7	7

(a) Viewing this as a game of investor versus economy and the above tables as a payoff matrix, what would the investor strategy of

$$[.3 \ .2 \ .5]$$

mean?

(b) What economy strategy corresponds to the economy experiencing medium growth?

(c) Given the investor and economy strategies in parts (a) and (b), what is the expected payoff to the investor?

(d) What is the optimal strategy for the investor, and what is the expected value of the game?

Solution:

(a) investing 30% of your money in bonds, 20% in stocks, and 50% in savings accounts

(b)

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

(c) 7.3% return

(d) The optimal strategy is to invest all of your money in savings account, and the expected value of the game is a 7% return.