Problem Set 1

Exercise Set 1.1

2) a) $x_1 - x_2 + x_3 = \text{sink}$ LINEAR, because sink = constant

b) $kx_1 - \frac{1}{k}x_2 = 9$ LINEAR

c) $2^k x_1 + 7x_2 - x_3 = 0$ LINEAR, since 2^k = constant

4) a) $\left[\begin{array}{ccc|c} 3 & -2 & -1 & \\ 4 & 5 & 3 & \\ 7 & 3 & 2 & \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{array}\right]$

c) $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & -1 & 1 & 1 \\ 0 & 3 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 7 & 0 & 1 \end{array}\right]$

d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

5) a) $2x_1 = 0$
$3x_1 - 4x_2 = 0$
[redacted] $x_2 = 1$

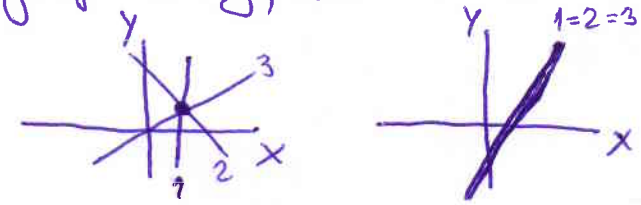
b) $3x_1 - 2x_3 = 5$
$7x_1 + x_2 + 4x_3 = -3$
$-2x_2 + x_3 = 7$

c) $7x_1 + 2x_2 + x_3 - 3x_4 = 5$
$x_1 + 2x_2 + 4x_3 = 1$

d) $x_1 = 7$
$x_2 = -2$
$x_3 = 3$
$x_4 = 4$

12.) $ax+by=k$
$cx+dy=e$
$ex+fy=m$

Saying that a system is consistent means that there is either 1 solution or infinite number of solutions.
Graphically, this looks like this:



If there is only 1 solution, taking out one of the lines leaves the solution unchanged, because the other two lines still intersect at the same point.
If there are infinite number of solutions, meaning all 3 eqns represent the same line, discarding one of the equations will not change the solutions, since the other two lines are exactly the same as the one removed.

13.) If $k=\ell=m=0$ we get

$ax+by=0 \longrightarrow y=-a/b\,x$
$cx+dy=0 \qquad y=-c/d\,x$
$ex+fy=0 \qquad y=-e/f\,x$

} point-slope form

The system is consistent when there is at least one solution to the system.
Here, the system must be consistent, because all three equations pass through the origin $(0,0)$, which is the point of intersection of the 3 lines, when the system has exactly 1 solution $(a/b \neq c/d \neq e/f)$